ARITHMETICK IN EPITOME:

OR, A

COMPENDIUM

Of all its

RULES,

BOTH

VULGAR *and* DECIMAL.

In Two PARTS.

To which are now Added,

Clear and plain DEMONSTRATIONS deduc'd from the Principles of *Arithmetick* itſelf; without either Reference to *Euclid*, or Uſe of *Algebra*.

By W. WEBSTER, Writing-Maſter.

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M.DCC.XXIX.

To the RIGHT HONOURABLE

Sir *Robert Walpole*,

Knight of the Moſt Noble Order of the Garter, Firſt Lord Commiſſioner of the Treaſury, and Chancellor of the Exchequer, *&c.*

SIR,

THO' the diſtruſt ſo natural to a young Author on his firſt Performance, did indeed diſcourage me from complying with my inclination, in offering this Attempt to your HONOUR on its firſt publication; yet being a little encouraged by the favourable reception its firſt Edition met with in the World, I did, with all humility, preſume to lay the ſecond Impreſſion at your Feet; well knowing the advantage it would appear with under the Patronage of a NAME ſo eminent for the encouragement of *Clerkſhip*, and of every other part of uſeful *Literature.*

Upon the third reviſal of this Work for the preſs, I made ſuch Additions and Improvements, as have, I hope, given it ſomewhat a better Title to ſo great a Protection: but ſtill its Imperfections, I am very ſenſible, will be but too apparent to your HONOUR's penetration; yet that Goodneſs, of which I have already had ſo abundant experience, gives me hopes your HONOUR will both excuſe what may remain amiſs, and forgive this renew'd preſumption of

Your HONOUR's

Moſt Obedient, and

Moſt Humble Servant,

William Webſter.

THE
PREFACE.

THE Indulgence with which the former Editions of this Treatiſe have been received, has encourag'd me to endeavour the making it ſtill more worthy the acceptance of the Publick; not only by correcting the many Errors which had before eſcaped, but alſo by enlarging it with very conſiderable Additions: having, in this Impreſſion, added the *Theory* to the *Practick Part*, and endeavour'd to inform the Reader's judgment, as well as to aſſiſt his memory.

But this I have attempted in a manner very different from the ordinary practice of Writers on this Subject, having never once quoted *Euclid*, nor perplex'd the Learner with (to him unintelligible) *Algebraical* Demonſtrations.

ARITHMETICK and GEOMETRY, are juſtly look'd upon as the two main pillars of the Mathematicks, upon which the whole Fabrick is rais'd; but in comparing them, they are too often confounded: for how great a ſimilitude ſoever there may be betwixt them, in ſome reſpects; yet, as their Subjects are different, ſo their Principles are certainly diſtinct, and each may be accounted for, without references to the other. Such compariſons may indeed be uſeful by way of Illuſtration, and give great ſatisfaction to thoſe who have a competent knowledge in both: But concluſions from *Geometrical* Demonſtrations, are beyond the reach of the young Arithmetician; and the Theory of *Lines* and *Diagrams*, as they are commonly made uſe of, can only ſerve to bewilder and confound him.

ALGEBRA, indeed, is not properly a Science diſtinct from Arithmetick, but is only a different method of Computation, perform'd by ſubſtituting *Letters* in the place of *Figures*, and expreſſing the ſeveral parts of the operation by *Symbolical Characters.* 'Tis true, great diſcoveries have been made in Numbers by this means, and Arithmetick owes many of its improvements to this admirable invention: beſides, it furniſhes us with a ſhort way of arguing, and brings the proofs of a Propoſition into a little compaſs. But however uſeful it may be to thoſe who underſtand it, it is certainly an unintelligible jargon to the mere Numeriſt, and can give him very little ſatisfaction when laid before him as Demonſtration.

Our knowledge in ſcience ſhould riſe gradually, from one ſtep to another, and our references ſhould be always backwards to what went before; but as the uſeful parts of Arithmetick have been commonly taught by practical Rules, which are uſually taken upon content, ſo little exactneſs is generally obſerv'd in the order of their delivery, as to their dependance upon each other: the reaſon of which is plain, for as all Affairs do not require the whole knowledge of Numbers, and as moſt common Buſineſs may be done without much skill in *Fractions*, ſo it is not only common, but reaſonable enough, to carry the Learner thro' whatever may be perform'd by *Whole-Numbers*, before he is entered upon the greater difficulties of *Fractional-Parts*. However, the gradation mentioned, ſhould be obſerved as much as poſſible, eſpecially in the buſineſs of Demonſtration, and the Maxims referred to for explanation, ſhould never be of a more abſtruſe nature than the thing to be explain'd.

I have therefore endeavour'd to give the inquiſitive Reader who would underſtand the reaſon of things, all the ſatisfaction I can, and have attempted to account for the Principles of Arithmetick, from ſuch ſelf-evident Propoſitions, and natural conſiderations, as flow moſt immediately from Numbers themſelves, and fall within the comprehenſion of ſuch who are the common conſulters of *Arithmetical Tracts*; and who, it may be preſum'd, are generally unacquainted with the more abſtruſe parts of the Mathematicks.

My firſt deſign in this Treatiſe was, according to its Title, to epitomize the Art; and to reduce to a *pocket Size*, what has ſometimes been ſwell'd to a *folio*. Indeed what I firſt publiſhed, was no more than a practical Memorandum, and conſequently conſiſted only of Rules and Examples, with few or no explications: but tho' the enlargements are now very conſiderable, I have not yet gone beyond my deſign; it is ſtill a Compendium, and far ſhort in bulk of other Treatiſes on the ſubject. I may therefore now recommend it to the ingenious *Clerks*, and *Accomptants* of the ſeveral Offices of *Great Britain* (for whoſe uſe eſpecially it was at firſt deſign'd) and to ſuch other *Gentlemen* whoſe Buſineſs requires the practice of Arithmetick, with more confidence than before; it being very little encreaſed in Size, tho' very much improv'd in Precept.

As I ſuppoſe my Reader acquainted with the manner of performing the common Rules of *Addition*, *Subſtraction*, *Multiplication*, and *Diviſion*; ſo I have made the Work ſhort, by omitting ſuch needleſs inſtructions: but the *Reaſons* of thoſe operations (being the Fundamentals of Arithmetick) I thought neceſſary to lay down. The *Rule of Three* I have alſo endeavoured ſo to explain, that the application of *Proportion*, as it runs thro' almoſt all the other Rules, may be fully underſtood.

And for thoſe practices of Arithmetick which will admit of no other than an *Algebraical* Demonſtration, being generally parts more curious than uſeful, ſuch as are ſome propoſitions in *Progreſſion*, and the *Double Rule of Falſe*; I have thought it ſufficient to give the practical Rules for their performance, without their Demonſtration; ſince thoſe who are proficients in *Algebra*, cannot be thought to want ſuch explanation; and thoſe who are not, cannot be ſuppoſed to underſtand it.

I have indeed ventured to account for the Extraction of the Roots, in a manner which ſeems ſomething contradictory to my profeſs'd deſign, but when it is conſidered that the Subject of thoſe operations is entirely *Geometrical*, though the Operations themſelves are *numerically* perform'd, it will be found that there is no other rational method of Demonſtration, but that of *Lines*; which, however, I hope, I have ſo laid down, as to be clear and intelligible to every Reader of common ſenſe, who but underſtands the difference betwixt a *Line*, a *Surface*, and a *Solid.*

As to the diſpoſition of the *Chapters*, they follow much in the ſame order as they are generally taught in *Schools*: for tho' I am ſenſible that the Rules of *Practice* are not thoroughly to be underſtood without ſome knowledge of *Vulgar Fractions*, yet I have choſe to place Fractions in the laſt Chapter of *Vulgar Arithmetick*, becauſe (as I before obſerv'd) moſt common Buſineſs may be done by Whole Numbers. So the ſpeculative Rules of *Progreſſion*, or *Arithmetical* and *Geometrical Proportion*, which in a juſt *Theoretick* way of treating the ſubject of Numbers, ought to precede the *Rule of Three*, I have thought better to place juſt before *Vulgar Fractions*: however, *Progreſſion* and *Vulgar Fractions*, being each diſtinct Chapters, the Reader may look into them whenever he thinks it moſt proper.

The Second Part of this Treatiſe which contains the Doctrine of *Decimal Fractions*, &c. is a kind of Arithmetick peculiarly, as it were, adapted to the concerns of *Gentlemen*: I have therefore been more large than ordinary upon that Subject, and have run over the ſeveral Rules again, to ſhew its particular uſe and application. In the Chapter of *Intereſt* and *Rebate*, its excellency will moſt appear, every Calculation of that kind being perform'd to great advantage by decimal Numbers. Under that head will likewiſe be found ſeveral uſeful *Tables* (which, that they might be perfectly correct, are exactly engrav'd on Copper) for the ready diſcovering the Amount, and preſent Worth both of ſingle Sums, and of Annuities. There are, beſides theſe, ſome other neceſſary *Tables* interſpers'd through this part of Arithmetick (which are alſo with the ſame exactneſs, printed from copper Plates) as thoſe for the reduction of the known parts of Coin, Time, Weight, and Meaſure, into decimal Parts of their proper *Integers*; and alſo that moſt uſeful *Table*, which ſhews the number of Days from any day in any Month, to the ſame day in any other. The *Contents* will ſhew where to find any of theſe things, and in the Work itſelf, are ſufficient explanations both of their formation and uſe. I ſhall therefore add nothing further by Way of Preface, but ſubmit the whole Performance to the judgment and candour of the Publick.

VULGAR ARITHMETICK,

IN

Whole Numbers and Fractions.

PART I.

AS it is inconſiſtent with the intended brevity of this Treatiſe, to enter upon a nice enquiry after the firſt Inventors of Arithmetick, or by what degrees it has been rais'd to its preſent perfection: ſo neither is it my deſign, in the following pages, to initiate abſolute Beginners; but ſuppoſing my Reader acquainted, at leaſt, with the methods of Adding, Subſtracting, Multiplying, and Dividing, (tho' perhaps ignorant of the Reaſons thereof) I ſhall lay before him, in as few words as poſſible, the Reaſons of thoſe Fundamentals, and their Application throughout all the varieties of practice.

Arithmetick (which is juſtly defined *The Art of Reckoning)* has for its ſubject Number; and teaches us to give proper anſwers to all ſuch queſtions as demand *How many?* For the more ready performing of which, the ten following Characters, or Figures, have been invented and agreed upon by the greateſt part of the known World, *viz.*

1 2 3 4 5 6 7 8 9 0.

And by the various combinations and repetitions of theſe ten Characters, may every Number, how great ſoever, be eaſily and expeditiouſly expreſs'd.

Thus much in general: I now proceed to particulars.

* * *

CHAP. I.

Of the five firſt and fundamental Rules, viz. *Numeration, Addition, Subſtraction, Multiplication, and Diviſion.*

NUMERATION, OF NOTATION.

BY Numeration, we learn the different value of Figures, by their different places; and, of conſequence, to read or write any Sum, or Number.

The TABLE.

9	Unites.	1
90	Tens.	12
900	Hundreds.	123
9000	Thouſands.	1234
90000	X Thouſands.	12345
500000	C Thouſands.	123456
9000000	Millions.	1234567
90000000	X Millions.	12345678
900000000	C Millions.	123456789

From this Table may be obſerv'd:

1. The names of the ſeveral places, *viz.* Unites, Tens, Hundreds, *&c.* which proceed (encreaſing by a ten-fold proportion) from the right hand to the left.

2. That every Figure hath two values, one in itſelf; the other, from the place it ſtands in. Thus, on the left ſide of the Table, the figure 9 in the upper line, ſtanding in the Unites place, is only nine; but in the ſecond line, being removed into the place of Tens, becomes ninety; and in the third line is nine hundred, *&c.*

3. That tho' a Cypher is nothing in itſelf, yet it gives value to other figures, by removing them into higher places.

All which being very obvious, I proceed to the next Rule.

ADDITION.

BY Addition, we find the whole, or total, of two or more parts, or ſums.

In ſetting down the numbers to be added, care muſt be taken to place every figure in its proper column; that is, Unites under Unites, Tens under Tens, *&c.* Then will the reaſon of the work (the manner of which, I ſuppoſe my Reader well acquainted with) appear very evident from this undeniable Maxim, *viz. That the Whole is equal to all its Parts.* And the method of ſetting down the total may eaſily be accounted for, from the nature of Numeration, which explains the different value of places, as they proceed from the right to the left hand: For as 9 is the greateſt ſimple character, or figure; ſo every number exceeding 9, being compound, muſt require more places than one to expreſs it. Thus the number 10 can no otherwiſe be expreſs'd in figures, but by removing the figure 1 into the place of Tens, which is done by ſupplying the Unites place with a cypher: And as it is the ſame with every other column (Ten being ſtill the Proportion of increaſe) conſequently, when the ſum of any column amounts to 10, or more, the unites exceeding, if there be any, or a cypher, if none, muſt be ſet under ſuch column; and the ten, or tens, of the amount carry'd on as ſo many unites, to the next column on the left.

What is here obſerv'd, as to carrying the Tens (the proportion of increaſe) from one column to another in Integers, may be as juſtly apply'd to the numbers we ſtop at, in adding ſums of different Denominations.

For your greater eaſe in caſting up of Money, learn the following Table:

PENCE-TABLE.

Pence.		*s.*	*d.*	*Pence.*		*s.*	*d.*
20	is	1	8	80	is	6	8
30		2	6	90		7	6
40		3	4	100		8	4
50		4	2	110		9	2
60		5	0	120		10	0
70		5	10				

Examples in Whole Numbers, and Money.

l.	*Yards.*	*l.*	*s.*	*d.*	
756	1325	735	18	09 ¼	
132	4532	423	10	10 ½	
458	7345	784	12	05 ¾	
736	1298	297	08	04	
857	8473	542	11	11 ½	
241	5249	298	14	07 ¾	
3180	28222	3082	17	00 ¼	*Total.*

Examples in Avoirdupoiſe *and* Troy-Weights.

AVOIRDUPOISE.

Tons.	C.	*q.*	lb.	*oz.*	*drams.*	
753	19	3	27	15	15	
347	08	1	17	10	06	
283	11	0	12	03	10	
549	05	2	13	09	07	
251	13	0	15	03	11	
2185	18	1	02	11	01	*Total.*

TROY-WEIGHT.

lb.	*oz.*	*dwts.*	*grs.*	
327	11	19	23	
429	10	17	19	
274	08	13	09	
478	10	12	05	
326	09	18	11	
1838	04	01	19	*Total.*
1510	04	01	20	*Tot. without the Top-line.*
1838	04	01	19	*Proof.*

To know how many Drams make an Ounce, or Grains a Penny-weight, *&c.* ſee the following Tables, Page 19.

Proof of Addition.

The proof of this Rule is uſually by a ſecond addition, without the top-line; which ſecond ſum, if, when added to the uppermoſt line, it makes the firſt total, the work is ſuppoſed right. *See the laſt Example.*

But it is as good, if not a better Proof, to produce the ſame total, by adding your columns both up and down.

SUBSTRACTION.

Subſtraction teaches us, by taking a leſſer number from a greater, to find the Remainder.

The reaſon of this Rule is evident from the ſame principles as Addition; Subſtraction being but the reverſe thereof: And the number borrowed in any column (like what we ſtop at in Addition) being always ſo many as would make 1 in the next.

Examples in Integers, and Money.

	Yards.		*l.*	*s.*	*d.*
	7146325	*Lent*	812	13	08 ¼
	1483972	*Paid*	190	19	10 ½
Rem.	5662353	*Rem.*	621	13	09 ¾
Proof.	7146325	*Proof*	812	13	08 ¼

	l.	*s.*	*d.*
Borrow'd at ſeveral Times.	429	11	08 ½
	212	17	10
	356	17	05 ¾
Borrow'd in all	999	07	00 ¼
Paid	519	18	09 ¾
Rem.	479	08	02 ½
Proof	999	07	00 ¼

AVOIR-

AVOIRDUPOISE.

	Tons.	*C.*	*q.*	*lb.*	*oz.*	*drams.*	
	92	10	0	07	03	13	
	14	13	3	11	12	05	
	77	16	0	23	07	08	*Rem.*
	92	10	0	07	03	13	*Proof.*

TROY-WEIGHT.

	l.	*oz.*	*dwts.*	*grs.*	
From	672	10	05	09	
Take	139	11	05	21	
	532	10	19	12	*Rem.*
	672	10	05	09	*Proof.*

Proof of Subſtraction.

Sums in this Rule are eaſily proved, by adding their remainders to their leſſer numbers; which (if right) will make the greater.

MULTIPLICATION.

MUltiplication (which is the fourth Rule) ſerves inſtead of many Additions; the product of a Multiplication being only the repetition of the Multiplicand ſo many times as there are unites in the Multiplier.

Note, The ready performance of this and the next Rule, entirely depends upon the perfect knowledge of the following Table.

The TABLE.

3 times	3	9
	4	12
	5	15
	6	18
	7	21
	8	24
	9	27
4 times	4	16
	5	20
	6	24
	7	28
	8	32
	9	36
5 times	5	25
	6	30
	7	35
	8	40
	9	45
6 times	6	36
	7	42
	8	48
	9	54
7 times	7	49
	8	56
	9	63
8 times	8	64
	9	72
9 times	9	81
12 times	2	24
	3	36
	4	48
	5	60
	6	72
	7	84
	8	96
	9	108
	10	120
	11	132
	12	144

Three terms are uſed in Multiplication, *viz.* *Multiplicand*, *Multiplier*, and *Product.*

The following Example ſhews to which line each name belongs.

EXAMPLE.

7563254138 *Multiplicand.*
23456789 *Multiplier.*

68069287242
60506033104
52942778966
45379524828
37816270690
30253016552
22689762414
15126508276

177409656468442882 *Product.*

1
8+8 *Proof.*
1

That Multiplication ſerves inſtead of many Additions; and, conſequently, that the truth of its operation depends upon the ſame Reaſons, is eaſily proved: For ſuppoſe it were required to know the ſum of four ſevens; by Addition, the work will ſtand thus:

7
7
7
7
28

Too tedious a method for practice; being diſcoverable at once, by the Table of Multiplication, to be 28.

Nothing therefore remains to be accounted for in this Rule, but the reaſon of placing every following particular product a place nearer to the left hand than the product foregoing: But regard being ſtill had to the nature of Numeration, it will plainly appear, that as we begin with the right-hand figure of the Multiplier, or Unites place; ſo the ſecond figure ſtanding in the place of Tens, the product thence ariſing muſt, of conſequence, be ten times greater than the ſame figure, ſtanding in the place of Unites, would have produced ; and ſo on, applying the ſame conſideration to the produce of all the other figures in the Multiplier.

Proofs of Multiplication.

Multiplication is uſually thus proved. Caſt out the *nines* from the Multiplicand and Multiplier, and place the remainders on the right and left ſides of a Croſs, thus made ┼. Theſe two figures multiply'd together, muſt have the *nines* caſt out of their product, and the remainder placed at top : Then caſting the *nines* alſo out of the product of your Multiplication, place its remainder at the bottom, which, if it agrees with the figure at the top, the work is ſuppoſed right. *See the Example.*

A more certain Proof ;

A Multiplication ſum is then right, when the product, divided by the Multiplier, quotes the Multiplicand ; or, divided by the Multiplicand, quotes the Multiplier.

ABBREVIATIONS.

(1*ſt.*) When either your Multiplicand, or Multiplier, or both, have one or more cyphers to the right hand, only multiply by the ſignificant figures, and ſet on the right hand of the product ſo many cyphers as were in both the Multiplicand and Multiplier.

EXAMPLES.

527000	7583
4000	7000
2108000000	53081000

(2*dly.*) Therefore any number may be multiply'd by 10, 100, 1000, *&c.* only by placing on the right hand of it, one, two, three, or more Cyphers: Thus, 7295 multiply'd by 10, is 72950; by 100, is 729500, *&c.*

(*3dly.*) To multiply any number by 5, add a cypher to it, and halve it. Or by 15, the ſame; and add both lines together.

EXAMPLES.

5334 *multiply'd*

by 5 —— 53340
is —— 26670

by 15 —— 53340
26670
is —— 80010

(*4thly.*) Any number may be multiply'd by 11, 111, or 112, *&c.* as in the following Examples.

EXAMPLES.

7136 *multiply'd by* 11.
7136
78496

by 111, *thus*: 7136
7136
7136
792096

And by 112, *thus*: 7136
7136
7136
7136
799232

Or thus: 7136
112
85632
7136
799232

We now proceed to our fifth Rule.

DIVISION.

BY Diviſion (which is the reverſe of Multiplication) we find how often one number is contain'd in another.

In this rule there are alſo four terms, the *Diviſor*, *Dividend*, *Quotient*, and *Remainder*.

EXAMPLE.

Diviſor. *Dividend.* *Quotient.*
3456789) 5678954367833 (164284
22221653
14809156
9820407
29068298
14139863
312707 *Remainder.*

As Multiplication ſerves inſtead of many Additions; ſo Diviſion ſupplies the place of many Subſtractions: as may be thus made evident. Suppoſe it were required to divide 28 by 7, that is, to find how often 7 is contained in 28; by Subſtraction the work will ſtand thus:

28
7
21
7
14
7
7
7
0

By which we find that 7 is 4 times contain'd in the number 28. But it may be diſcover'd by the rules of Diviſion at one trial.

There are many methods of working this rule. The following example is divided ſix ſeveral ways.

Firſt.

$\cancel{1}\cancel{0}(2$
$\cancel{1}\cancel{0}\cancel{0}8(1$
$\cancel{4}\cancel{2}\cancel{6}\cancel{4}\cancel{5}(1332$
$\cancel{3}\cancel{2}\cancel{2}\cancel{2}\cancel{2}$
$\cancel{3}\cancel{3}\cancel{3}$

Second.

$\cancel{1}\cancel{0}(2$
$\cancel{1}\cancel{0}\cancel{0}8(1$
$32)\cancel{4}\cancel{2}\cancel{6}\cancel{4}\cancel{5}(1332$
$\cancel{3}\cancel{2}\cancel{6}\cancel{6}\cancel{4}$
$\cancel{9}\cancel{9}\cancel{6}$

Third.

$\cancel{1}\cancel{0}(2$
$\cancel{1}\cancel{0}\cancel{0}8(1$
$32)\cancel{4}\cancel{2}\cancel{6}\cancel{4}\cancel{5}(1332$

Fourth.

21
———
·8
10
10
32)42645(1332
32
96
96
64

Fifth.

32)42645(1332
32
———
106
96
———
104
96
———
·85
64
———
21
———

Sixth.

32)42645(1332
———
106
———
104
———
·85
———
21

Note, The two laſt, call'd the *Italian* ways, are moſt generally uſed.

Mr. *Alingham* has, indeed, ſet down nine ways; but his three others are ſo very like the 1ſt, 2d, and 3d of theſe, that they can ſcarce be call'd different.

ABBREVIATIONS.

(*1ſt.*) If there are any cyphers on the right hand of your Diviſor, you may cut off ſo many cyphers, or figures, on the right hand of your Dividend; but remember to bring them down (if figures) to the remainder.

EXAMPLE.

21|00) 8645|29(411
84

24
21

35
21

1429

(*2dly.*) By the foregoing rule, you may obſerve, that to divide by 10, 100, 1000, *&c.* is only to cut ſo many figures from the right hand of the Dividend, as there are cyphers in the Diviſor.

EXAMPLE.

1|000) 43682|735 (
So the Quotient is 43682, *the Remainder* 735

(*3dly.*) When your Diviſor is 12, or conſiſts only of one ſingle figure, or can be reduced to one, by cutting off cyphers from its right hand, the work may be eaſily perform'd in one line, thus:

RULE.

Drawing a line under the Dividend, ſet down under its figure, how often the Diviſor is contain'd in it; what remains, imagine placed before the next figure; and conſidering how often your Diviſor is contain'd in the ſum it makes, ſet down the number underneath, as before; and ſo proceeding through all the figures, ſet down what remains at laſt, in the place where your Quotient uſed to ſtand.

EXAMPLES.

4)93645(1	12)83675(11	7|00)5635|15(
23411	6972	805

If you are to divide ſeveral numbers by one common Diviſor (as in the calculating of Tables, *&c.*) that you may know exactly at once how often your Diviſor will go, in ſome convenient corner, make a Table of your Diviſor, by multiplying it ſeverally by all the nine digits: Thus, ſuppoſe 562 your Diviſor:

562	1
1124	2
1686	3
2248	4
2810	5
3372	6
3934	7
4496	8
5058	9

Proofs of Diviſion.

(*1ſt.*) Multiplication and Diviſion mutually prove each other: For as if you divide the Product of a Multiplication by the Multiplier, the Quotient will be the Multiplicand; ſo, if you multiply the

Quotient of a Diviſion by the Diviſor (taking in the remainder) the product will be the Dividend.

(2*dly.*) Another proof of Diviſion is, by adding together thoſe lines in the following example, mark'd with Aſteriſms, (being the particular products of the Diviſor, multiply'd ſeverally by each figure in the Quotient, together with the remainder of the Diviſion) the total of which (if right) will be the Dividend.

(3*dly.*) Diviſion may alſo be proved as Multiplication, by a Croſs, thus; caſting out the nines from the Diviſor and Quotient, place the remainders on its right and left ſides; then multiplying the two figures ſo placed together, and caſting the nines from the product, add what's left to the remainder of the Diviſion; and ſtill caſting out the nines, let the overplus be placed at the top; then alſo caſting the nines from the Dividend, ſet down the figure remaining at the bottom, which if it agrees with that at top, the work may be ſuppos'd right. See each proof in the following examples.

EXAMPLE.

	736)863256(	1172
	736*	736
3	1272	7032
7 ┼ 2	736*	3516
3		8204
3d Proof.	5365	
	5152*	862592
		664 *Remainder.*
	2136	
	1472*	863256 *1ſt Proof.*
	664*	
	863256 *2d Proof.*	

Before we proceed to *Reduction*, it will be proper to inſert the following Tables.

TABLES *of* Engliſh *Coins, Weights and Meaſures.*

Firſt of COINS.

ACCOMPTS are kept in Pounds, Shillings, Pence, and Farthings thus divided.

4 Farthings	make	1 Penny,	thus mark'd	*d.*
12 Pence	make	1 Shilling,	thus mark'd	*s.*
20 Shillings	make	1 Pound,	thus mark'd	*l.*

But the uſual Coins are,

			l.	*s.*	*d.*
Of Gold,	A Jacobus,	Value	1	5	0
	A Carolus,	Value	1	3	0
	A Guinea,	Value	1	1	0
	A ½ Guinea,	Value	0	10	6
Of Silver,	A Crown,	Value	0	5	0
	A ½ Crown,	Value	0	2	6
	The names of the reſt ſpeak their value, as a Shilling, a Six-pence, a Groat, or 4 *d.* a Three-pence, a Two-pence, a Penny.				
Of Copper,	A ½ Penny,	thus writ,	0	0	0½
	A Farthing,	thus writ,	0	0	0¼

Beſides the above-mention'd, we have ſtill in uſe the names of ſome other pieces, which are now but imaginary, *viz.*

		l.	*s.*	*d.*
A Mark,	Value	0	13	4
An Angel,	Value	0	10	0
A Noble,	Value	0	6	8

WEIGHTS.

TROY.

24 Grains	make	1 Pennywt.	thus mark'd	*dwts.*
20 Pennywts.		1 Ounce,		*oz.*
12 Ounces		1 Pound,		℔.

By *Troy-Weight* are weighed Jewels, Gold, Silver, Corn, Bread, and all Liquors.

APOTHECARIES.

20 Grains	make	1 Scruple,	thus mark'd	℈
3 Scruples		1 Dram,		ʒ
8 Drams		1 Ounce,		℥
12 Ounces		1 Pound,		℔

By theſe Weights, Apothecaries compound their Medicines; but buy and ſell their Drugs by *Avoirdupoiſe.*

AVOIRDUPOISE.

16 Drams	make	1 Ounce,	thus mark'd	*oz.*
16 Ounces		1 Pound,		℔
28 Pounds		1 quarter of a Hund.		*qrs*
4 Quarters		1 Hundred,		C.
20 Hundred		1 Ton,		*ton*

This is, at preſent, the common weight of *England*, by which, Butter, Cheeſe, and all Groceries, *&c.* are weighed.

Note, One pound *Avoirdupoiſe* is equal to 14 *oz.* 11 *dwts.* 15 *grs.* ½ *Troy*; and one ounce *Troy* is equal to 1 *oz.* 1 *dram*, and ſomething above ½, *Avoirdupoiſe.*

WOOL-WEIGHT.

7 Pounds	make	1 Clove,	mark'd as written.
2 Cloves		1 Stone,	
2 Stones		1 Todd,	
6 Todds and ½		1 Wey,	
2 Weys		1 Sack,	
12 Sacks		1 Laſt,	

LEAD.

C.
19 ½ makes a Fodder.

MEASURES.

WINE.

2 Pints	make	1 Quart,	thus mark'd	*Qrt.*
4 Quarts		1 Gallon,		*Gall.*
63 Gallons		1 Hogſhead,		*Hdd.*
2 Hogſheads		1 Pipe,		*Pipe.*
2 Pipes		1 Tun,		*Tun.*

BEER and *ALE.*

2 Pints	make	1 Quart,	thus mark'd	*Qrt.*
4 Quarts		1 Gallon,		*Gall.*
9 Gallons		1 Firkin,		*Firk.*
2 Firkins		1 Kilderkin,		*Kild.*
2 Kilderkins		1 Barrel,		*Bar.*
3 Barrels		1 Butt,		*Butt.*

Note, 8 Gallons make 1 Firkin of Ale.

DRY.

8 Pints	make	1 Gallon	thus mark'd,	*Gall.*
2 Gallons		1 Peck,		*Peck.*
4 Pecks		1 Buſhel,		*Buſh.*
4 Buſhels		1 Coom		*Coom.*
2 Cooms		1 Quarter,		*Qr.*
5 Quarters		1 Wey,		*Wey.*
2 Weys		1 Laſt,		*Laſt.*

LONG.

3 Barley-Corns	make	1 Inch,	mark'd as writ
12 Inches		1 Foot,	
3 Feet		1 Yard, *	
5 Yards and ½		1 Pole or Perch,	
40 Poles		1 Furlong,	
8 Furlongs		1 Mile,	

* 1760 Yards make a Mile.

LAND.

40 Square Perches	make	1 Rood.
4 Roods		1 Acre.

Note, That a Geometrical Pace is 5 Feet, and that there are 1056 ſuch paces in an *Engliſh* Mile.

CLOTH.

4 Nails	make	1 Quarter,	mark'd	*qr.*
4 Quarters		1 Yard,		*yd.*

Note, An Ell *Flemiſh* is 3 *qrs* of a Yard; an Ell *Engliſh* is 5.

12	make	1 Dozen.
12 Dozen		1 Small Groſs.
12 Small		1 Great Groſs.

CHAP. II.

Of REDUCTION.

REduction is but an application of Multiplication and Diviſion: For,

Firſt, All great names are brought into ſmall, by multiplying with ſo many of the little ones as make one of the great.

Secondly, All ſmall names are brought into great, by dividing by ſo many of the little ones as make one of the great.

Thirdly, To change one ſort of money, or weight, *&c.* into another, is only to bring both into one name, and to divide the one by the other.

EXAMPLES of each Sort.

(Firſt Sort.)

l. *s.* *d.*
In 4295 12 3 how many Farthings?
'tis multiply'd by 20 *becauſe* 20 s. *make* 1 l. Note, *The* 12 s. *are taken in.*
———
85912
'tis multiply'd by 12 *becauſe* 12 d. *makes* 1 s. *And here the* 3 d. *are added.*
———
1030947
'tis multiply'd by 4 *becauſe* 4 *Farthings make* 1 *Penny.*
———

Anſwer. 4123788 *Farthings.*

(*Second Sort.*)

In 4123788 Farthings, how many Pounds?

4)4123788(

12)1030947(3

2|0)8591|2(

4295 *l.* 12 *s.* 3 *d.* *facit.*

We divide here by 4, 12, and 20, for the ſame reaſons that we multiply'd by them in the firſt example.

(*Third Sort.*)

s. *d.*

Change 45967 *French* Crowns, at 4 6 each, into Guineas.

2

Six-pences in a French *Crown.* } 9

s.
21
2

42 } *Six-pences in a Guinea,*

45967
9

42)413703(9850 *Guineas.*
378

357
336

210
210

03 *Six pences.*

More EXAMPLES.

In 85647 Guineas, how many Pounds Sterling?
Anſwer, 89929 *l.* 7 *s.*

In 59463 Marks, at 13 *s.* 4 *d.* how many Pounds?

Anſwer, 39642 *l.*

How many Dollars, at 4 *s.* 4 *d.* are there in 89573 Piſtoles, at 17 *s.* 6 *d?*

Anſwer, 361737 Dollars, and 6 *d.* remain.

In 82 lb. 9 *oz.* 10 *dwts.* 5 *grs.* how many Grains?

Anſwer, 476885 Grains.

In 56 *Tons*, 13 C. 2 *qrs.* 21 lb. 7 *oz.* 11 *Drams*, how many Drams?

Anſwer, 32505211 Drams.

How many minutes ſince the birth of our Saviour, it being 1729 years?

Anſwer, 908762400 minutes.

How many Gallons are there in 7569 Tuns?

Anſwer, 1907388 Gallons.

Note, Theſe queſtions revers'd, will make as many more good examples.

CHAP. III.

Of PROPORTION, *or the* RULE *of* THREE, *Direct*, *Indirect*, *and Double.*

THIS Rule is call'd the RULE of THREE, becauſe by three numbers given we find a fourth number ſought; which, when the Proportion is direct, muſt always bear the ſame ratio, or proportion, to the third number, as the ſecond bears to the firſt.

The chief difficulty of this Rule, lies in ſtating its queſtions. For your direction therefore obſerve, that of the three given numbers, two always contain a ſuppoſition, and the third a demand.

The number then on which the demand lies, muſt always be the third in your ſtating; of the other two, you will be ſure to find one of the ſame quality with the ſaid third; which being made your firſt, the number left, muſt, of conſequence, fall in the ſecond place, which ſecond number will alſo always be of the ſame kind with the fourth, or number ſought.

The queſtion being thus ſtated, you muſt (if they are not already ſo) bring your firſt and third numbers into one name, and your ſecond (if of ſeveral denominations) into its loweſt term; then multiplying your ſecond and third numbers together, and dividing the product by your firſt, the quotient will be the anſwer of your queſtion, in the ſame denomination you left your ſecond number. *See the Examples.*

EXAMPLES.

(*Ex.* 1*ſt.*) How many Ounces may I buy for 8 *l.* if I give after the rate of 2 *l.* for 18 Ounces?

If

l. *oz.* *l.*
If 2 *buy*——18——*what will* 8 ?
8
——
2)144(
——
Anſwer.—72 *oz.*

(*Ex.* 2*d.*) What muſt I give for 112 Ells of Holland, if I pay after the rate of 34 *l.* for 138 Ells ?

Ells. *l.* *Ells.*
If 138 *coſt*——34 *l.*——*what will* 122 ?
34
——
488
366
—— *l.* *s.* *d.*
138)4148(30 1 1¼
414
——
. . . 8
20
——
138)160(1
138
——
·22
12
——
138)264(1
138
——
126
4
——
138)504(3
414
——
·90

The Remainder 8, *being Parts of a Pound, are multiply'd by* 20, *to ſee what Shillings they will produce ; and ſo the other Remainders by* 12, *and* 4, *to bring out the odd Pence, and Farthings.*

This way of valuing the Remainder, is more fully explain'd in the ſeventh ſort of Reduction of Vulgar Fractions.

(*Ex. 3d.*) How many yards of Muſlin can I buy for 42 *l.* 12 *s.* if $2\frac{1}{2}$ yards come to 19 *s.* 6 *d?*

s. d.	*yards.*	*l. s.*
If 19 6 buy——	$2\frac{1}{2}$——*what will*	42 12
2	2	20
39	5	852
		2
		1704
		5
2)218(		39)8520(218 *facit.*
109 *yards.*		78
		·72
		39
		330
		312
		·18

PROOF.

Of every four numbers in direct proportion, the product of the two means multiply'd into each other, will be equal to the product of the two extreams ſo multiply'd: therefore, multiplying your firſt by your fourth number found, and comparing it with the produce of your ſecond by your third, if the products agree, the work is right. Thus, in the firſt example, 72 (the fourth number found) multiply'd by 2 (the firſt number) gives 144, equal to the produce of 8 times 18, the ſecond and third numbers.

And from hence ariſes the invention of the foregoing rule, for finding the fourth number; for if the ſecond number, multiply'd by the third, be equal to the firſt multiply'd by the fourth, it is plain that if the product of the ſecond and third be divided by the firſt, the Quotient muſt be the ſaid fourth number; becauſe every Dividend muſt be equal to the produce of its Diviſor and Quotient, the remainder (if any) being alſo conſider'd.

Sums in this rule may alſo be proved by a back-ſtating; thus reverſing the third Example.

```
    l.  s.              yards.         s. d.          yards.
If  42  12  will buy    109,  then     19  6  will buy  2½
    20                   39             2
   ----                 ----          ----
   852                  981            39
     2                  327
   ----                 ----
  1704             1704)4251(2½ yards.
                        3408
                        ----
                        ·843
                           2
                        ----
                        1686
                          18 { Remainder of the other
                        ---- { ſtating added.
                   1704)1704(1
                        ----
                        ···0
```

Though in the former of the foregoing proofs, the manner of working the *Rule of Three* is ſufficiently accounted for, and the reaſon of the operation plainly enough laid down; yet as this may juſtly be look'd upon as the main Rule of Arithmetick, and as the ſame application of proportion runs thro' almoſt all its other branches; it may, perhaps, help to make it ſtill more evident, to look back again upon the firſt example, and obſerve it in another light. The queſtion there ſtated was, *If* 2 *l.*—18 *oz.*—8 *l.* Now had it been, *If* 1 *l.*—18 *oz.*—8 *l.* 'tis clear,

lear, that if 1 *l.* would buy 18 *oz.* that then 8 *l.* would buy 8 times as many, that is 144 *oz.* But then, as this is upon the ſuppoſition that 1 *l* is the price of 18 *oz.* ſo if the price of the 18 *oz.* is doubled, that is, made 2 *l.* 'tis plain 144 *oz.* will then be twice the quantity that 8 *l.* will buy, and muſt therefore be divided by 2, to give the true anſwer. Again, had the ſuppoſed price of 18 *oz.* been 4 *l.* then would 8 *l.* have bought but a fourth part of 144 *oz.* *viz.* 36 *oz.* conſequently, in what proportion ſoever the firſt number encreaſes, in the ſame proportion muſt the fourth number decreaſe; which is plainly effected by dividing by the firſt number.

Note, In the operation of the *Rule of Three*, the firſt and third numbers, after preparation, are no more regarded, as of any denomination; but are multiply'd and divided with, only as abſolute numbers, encreaſing and decreaſing the middle number in juſt proportion.

Note alſo, That what has been ſaid of the reaſon of working the *Rule of Three*, ought alſo to be conſider'd in the operations of *Intereſt*, *Rebate*, *Fellowſhip*, *Exchange*, and all other applications of proportion.

We now proceed to ſome other practical obſervations.

1. If you would know at what rate you muſt ſell out your goods by retail, ſo as to make a propos'd gain by the whole, add the money you would gain to the ſum the whole goods coſt you; and then ſtate your queſtion thus: *If the whole be ſold for the total of the Coſt and Gain, what muſt any part?*

EXAMPLE.

Suppoſe I would, by the ſelling of 32 yards of Broad Cloth, which coſt me 40 *l.* gain 5 *l.* for what muſt I ſell it *per* yard? Then,

If

```
           yards.          l.            yard.
           If 32 be ſold for 45, what will 1?
    l.                     1
    40 Coſt.               — l.  s.  d.
     5 Gain.           32)45(1   8   1½ facit.
    —                      32
    45 Total.              —
                           13
                            20
                        ————————
                        32)260(8
                           256
                        ————————
                             4
                            12
                        ————————
                        32)48(1
                           32
                        ————————
                           16
                            4
                        ————————
                        32)64(2
                           64
                        ————————
                            0
```

(2*d.*) Or if damage having happen'd to the Cloth, 5 *l.* were to have been loſt by the whole, then the ſaid 5 *l.* muſt have been ſubſtracted from the coſt, and the remainder made the ſecond number, as before.

(3*d.*) If you would barter or exchange your goods for others, firſt find the value of your own, and then ſee what quantity of the others the ſum will purchaſe.

EXAMPLES.

EXAMPLES.

What quantity of Pepper, at 3 *s.* 6 *d. per* lb. may have in exchange for 426 C. of Tobacco, at 53 *s.* er C?

C. s. C.
Firſt, *If* 1 —— 53 —— 426
53

1278
2130

Anſwer, 22578 *Shillings.*

s. d. lb. s.
Then, *If* 3 6 —— 1 —— 22578
2 2

7 7)45156(6

Anſwer, 6450 *Pounds* 6/7

ABBREVIATION.

The work of ſome ſtatings may be much ſhortned, by dividing their firſt and ſecond, or firſt and third numbers, by any figure, ſo that nothing may remain.

EXAMPLE.

EXAMPLE.

l. *Men* *l.*

If 192|00 *would pay* 5|00 *how many would* 17664 *pay?*

32 2944

8 736

5

8)3680(

Anſwer, 460 *Men.*

The numbers of the above-ſtating are abbreviated, by firſt cutting off the Cyphers from the firſt and ſecond numbers, and then dividing the firſt and third numbers by 6 and 4.

The reaſons of this abbreviation may be ſeen in the fifth ſort of Reduction of Vulgar Fractions.

More Queſtions in the DIRECT RULE *of* THREE.

The cloathing of a Regiment of 740 men, comes to 3000 *l.* how much is that for each man?

Anſwer, 4 *l.* 1 *s.* 0 *d.* $\frac{3}{4}$

How long ſhall I be laying up 10000 *l.* if I put by $\frac{1}{2}$ a Guinea a week?

Anſwer, 366 years, 15 weeks, 4 days.

500 Sea-men are to have 4 *d.* $\frac{1}{2}$ *per* day each, what will pay them for 23 months?

Anſwer, 6037 *l.* 10 *s.*

What will an eſtate of 4000 *l. per Annum*, allow a Gentleman to ſpend a day?

Anſwer, 10 *l.* 19 *s.* 2 *d.*

A Gentleman's daily expences are 13 *s.* 7 *d.* above which, he yearly lays up 600 Nobles, at 6 *s.* 8 *d.* each,

ach, what is his Eſtate worth *per Annum?*

Anſwer, 447 *l.* 17 *s.* 11 *d.*

A man owing 736 *l.* 10 *s.* compounds with his Creditors for 7 *s.* 9 *d. per l.* what will that amount o ?

Anſwer, 285 *l.* 7 *s.* 10 *d.* ½

If at 5 *s. per* ell, I gain 8 *l. per Cent.* by my Cloth, what ſhall I gain *per Cent.* if I ſell the ell at 6 *s.* 3 *d?*

Anſwer, 10 *l.*

If Tobacco, which coſt 13 *d. per* lb, be ſold for 18 *d. per* lb, what is gain'd *per Cent?*

Anſwer, 38 *l.* 9 *s.* 2 *d.* ¾.

Bought 5 pieces of Holland, each containing 56 ells *Flemiſh*, at 3 *s.* 2 *d. per* ell, what ſhall I gain in the whole, if I ſell it for 5 *s.* 8 *d. per* ell *Engliſh?*

Anſwer, 3 *l.* 5 *s.* 4 *d.*

I have by me 96 lb. of Cloves, which coſt me 58 *l.* but ſome damage having happen'd to them, I am willing to loſe 8 *l.* in the whole; at what rate muſt I ſell them *per* ounce?

Anſwer, 7 *d.* ¾.

A Merchant ſends over to *France* 482 tons of Lead, at 4 *l.* 10 *s. per* Fother, *i. e.* 19 ½ *C.* what quantity of Wine, at 30 *l. per* pipe, may he expect in return?

Anſwer, 74 pipes, 19 gallons.

A Merchant ſends to *Spain* 1300 pieces of Broad Cloth, each piece 47 yards, at 15 *s.* 6 *d. per* yard, to have returns from thence; the one half in Wine, at 65 *l. per* Ton; the other half in Oranges, at 3 *l.* 10 *s. per* Cheſt; what quantity of each will he have?

Anſwer, { 364 Tons, 1 Hogſhead of Wine.
6764 Cheſts of Oranges.

Two men part, at the ſame time, from the ſame place; the one travels north 32 miles a day; the other, 36 miles a day ſouth; how long will it be before they are 2000 miles aſunder?

Anſwer, 29 days, 9 hours, 52 minutes.

The

The Indirect RULE *of* THREE.

IN the *Indirect Rule of Three,* the numbers are i reciprocal proportion, that is, the fourth number to be found, is to bear the ſame ratio to th ſecond as the third does to the firſt, but in an inverted order; that is, the greater the third term is in reſpect to the firſt, the leſs muſt the fourth b in reſpect to the ſecond.

This rule differs in its operation, from the Direct, in that, after the queſtion is ſtated, and th numbers of the ſtatings prepared, (as in the Direct Rule) your firſt and ſecond muſt be multiply'd together, and your third number be your Diviſor. The Quotient, as before, will be the anſwer.

EXAMPLES.

(*Ex. 1ſt.*) What number of men muſt be employ'd, to finiſh, in 12 days, what 43 men would be 35 days about?

days *men* *days.*
If 35 —— 43 —— 12
43

105
140

12)1505(5

Anſwer, 125 *Men.*

(*Ex. 2d.*) How many yards of Stuff 3 *qrs* wide, will hang a Room which requires 420 yards of 5 *qrs.* wide?

qrs. *yards.* *qrs.*
If 5 ——420—— 3
5
———
3)2100(
———
Anſwer, 700

The reaſon of this operation will appear plain (after what has been ſaid in the Direct Rule) by conſidering the laſt example. Now it is clear, that if the Stuff, being 5 quarters wide, there are 420 yards requir'd, then were the Stuff but 1 quarter wide, 5 times 420 yards, *viz.* 2100 yards, muſt be allowed; conſequently, if the Stuff be 3 quarters wide, one third part of thoſe yards will be ſufficient; therefore 2100 divided by 3, will give the true anſwer requir'd, *viz.* 700 yards.

To know whether a Queſtion belongs to the Direct *or* Indirect Rule *of* Three.

Obſerve, If the third number being more than the firſt number, requires more; or being leſs, requires leſs, 'tis *Direct*; but if the third number, being more, requires leſs; or being leſs, requires more, 'tis *Indirect.*

Or, without any regard to the diſtinction of *Direct* and *Indirect*; if *more* is required, let the leſſer of the two extreams be the Diviſor; if *leſs*, the greater.

More Queſtions in the Indirect Rule *of* Three.

If I lend *A.* 136 *l.* for 3 months, how long muſt I keep 42 *l.* of his, to requite my ſelf?

Anſwer, 9 months, 2 weeks, 6 days.

If 46 Clerks in 32 days finiſh a piece of writing, in what time would 55 Clerks accompliſh the ſame?

Anſwer, 26 days, 18 hours, 19 minutes.

A Garriſon, conſiſting of 1539 men, being beſieged, have proviſions only for 12 days; but it being neceſſary they ſhould hold out 3 weeks, how many men muſt be ſent out?

Anſwer, 660 men.

The DOUBLE RULE *of* THREE.

QUeſtions in this Rule have five Numbers propoſed, and are generally anſwered by two ſtatings; of which, one ſometimes happens to be *Indirect*, as in the ſecond Example.

EXAMPLES.

(*Ex.* 1*ſt.*) The carriage of 32 hundred weight 56 miles, comes to 12 *s.* After the ſame rate, what muſt I pay to have 78 hundred weight carry'd 94 miles?

```
            C.        s.        C.
First, If 32 ———— 12 ———— 78
                          12
                          ——— s. d.
                      32)936(29 3 facit.
                          64
                          ———
                          296
                          288
                          ———
                            8
                           12
                          ———
                       32)96(3
                          96
                          ———
                           0

        Miles.  s.  d.  Miles.
Then, If 56 ——— 29  3 ——— 94
                12
                ———
                351
                 94
                ————
                1404
               3159
               —————
           56)32994(589          12(589(1
              280                ———————
              ———                2|0|4|9(
              499                ————
              448          Anſwer, 2 l. 9 s. 1 d
              ———
               514
               504
               ———
                10
```

Note,

Note, It had been the ſame thing, if the miles had been made the firſt and ſecond numbers of the firſt ſtating; and the C. weights the firſt and ſecond numbers of the laſt.

Note alſo, It may be done in one ſtating, thus:

```
        C.        s.       C.
    If 32 ——— 12 ——— 78
       56 Miles.        94 Miles.
       ———              ———
       192              312
      160              702
      ————             ————
      1792             7332
                         12
                     ————— s. d.
              1792)87984(49 1
                    7168     or
                  ———————  l. s. d.
                   16304   2. 9  1
                   16128
                  ———————
                    ..176
                       12
                  ———————
              1792)2112(1
                   1792
                  ——————
                   ·320
```

(*Ex.* 2*d.*) How many men muſt be employ'd to reap 420 Acres in 17 days, if there were requir'd 37 men to reap 54 Acres in 5 days?

Acres. *Men.* *Acres.*
Firſt, *If* 54——37——420
37

2940
1260

15540

54)15540(287
108

474
432

420
378

42

Days. *Men.* *Days.*
Then, *If* 5——287——17
5

17)1435(84
136

75
68

Anſw. 84 *Men.* 7

Note, If you would work ſuch queſtions of the *Double Rule of Three*, as have one of their proportions indirect, by one ſtating; you muſt multiply the third number of your ſtating, by that number you would otherwiſe have placed under your firſt; and your firſt number by that you would have placed under the third; as in the following Example.

EXAMPLE.

Acres. *Men.* *Acres.*
If 54——37——420 } *The number of day*
*17 5 } *which have relation to th*
—— —— } 54 *Acres.*
378 2100
54 37
—— ——
918 14700
6300
—— *Men.*
918)77700(84 *anſwer, as before.*
7344
——
4260
3672
——
588

* *The number of days which have relation to the* 420 *Acres.*

Dr. *Harris*, in his *Lexicon Technicum*, teaches another way of ranging the five Numbers given in a *Double Rule of Three* Queſtion; which, according to the following Rules, gives the ſolution, whether the proportion be direct or indirect. Take his method in his own words.

I. "Obſerving, that the given terms are always "five, whereof three are conditional and antece-"dent, or ſuppoſitious, the other two demand the "queſtion, and are conſequents anſwering ſome of "the former antecedents; inſomuch, that with "the anſwer there will be as many conſequents as "antecedents, which muſt match one another in "the ſame denomination exactly.

II. "For the right placing of the queſtion and "terms, the three terms of the conditional part "are duly to be regarded: Let that which is the "principal cauſe of Loſs and Gain, Increaſe or "Decreaſe, Action or Paſſion, be put in the first "place; and that which betokeneth the ſpace of

" time, diſtance of place, *&c.* be put in the ſe-
" cond place; and the remaining part in the third.
" The conditional part thus ſtated, the other two
" terms, wherein the demand lies, muſt be placed
" ſo under the former terms, that they may cor-
" reſpond one with another.

RULE 1.

" Then, if the blank, or place ſought, fall un-
" der the third term, multiply the three laſt terms
" for a Dividend, and the two firſt for a Diviſor;
" and the Quotient gives the ſixth term required.

RULE 2.

" But if the blank fall under the firſt or ſecond
" term, multiply the 1ſt, 2d, and 5th terms for a
" dividend, and the 3d and 4th for a Diviſor:
" The Quotient gives the anſwer.

EXAMPLE 1.

" If 12 Rods of ditching be done by 2 men in 6
" days, how many Rods ſhall be wrought by 8 men
" in 24 days?

Anſwer, 192.

The numbers being ſtated as before directed, they will ſtand thus, the blank falling under the third place.

Men.	*Days.*	*Rods.*
2	6	12
8	24	

Therefore, by the firſt Rule, the three laſt terms, *viz.* 12, 8, and 24, being multiply'd into each other, give 2304 for a Dividend; and the two firſt terms, *viz.* 2 and 6, multiply'd together, give 12 for a Diviſor, which quotes 192, the anſwer.

EXAMPLE 2.

If 2 men work 12 Rods in 6 days, how many men will work 192 Rods in 24 days?

Men	*Days*	*Rods.*
2	6	12
	24	192

Here, according to the ſecond Rule, the firſt, ſecond, and fifth terms multiply'd, give 2304, which divided by 288, the product of the third and fourth numbers, quotes 8, the anſwer.

More *EXAMPLES.*

If 400 pecks of Corn will ſerve 32 horſes 108 days, what quantity will 500 horſes eat in 20 days?

Anſwer, 1157 *pecks,* ¼

If 56 gallons of Drink ſerve 25 perſons 120 days? how long will 200 gallons ſerve 12 perſons?

Anſwer, 892 *days.*

CHAP. IV.

Of PRACTICE, *and* TRETT *and* TARE.

BY PRACTICE, Merchants compendiouſly caſt up the price of their Commodities.

And by TRET and TARE, deduct their Allowances.

Both theſe Rules will be beſt explain'd by examples; and both require the perfect knowledge of the following

TABLE.

The even Parts.

Of a Pound.

s.	d.	
1	0	1/20
1	8	1/12
2	0	1/10
2	6	1/8
3	4	1/6
4	0	1/5
5	0	1/4
6	8	1/3
10	0	1/2

The even Parts.

Of a Shilling.

d.	
1	1/12
1½	1/8
2	1/6
3	1/4
4	1/3
6	1/2

Of a Penny.

Far.	
1	1/4
2	1/2

The even Parts.

Of a Ton.

C.	
2	1/10
2½	1/8
4	1/5
5	1/4
10	1/2

Of a Hund.

lb.	
14	1/8
16	1/7

Of ½ C.

lb	
7	1/8
8	1/7
14	1/4

Before we proceed to the rules of Practice, it will be proper to ſhew the manner of multiplying and dividing numbers of ſeveral denominations, by a ſingle figure, without Reduction.

Example of MULTIPLICATION.

l. *s.* *d.*
Multiply 5 8 4 *by* 8.
8

facit 43 6 8

Here, firſt the 4 *d.* multiply'd by 8, gives 32 *d.* equal to 2 *s.* 8 *d.* therefore ſetting down 8 in the place of pence, we carry 2 to the produce of the ſhillings: Again, 8 times 8 *s* being 64 *s.* and the 2 carry'd making it 66 *s.* or 3 *l.* 6 *s.* the 6 is ſet down in the place of ſhillings, and the 3 carry'd on to the produce of the pounds. Laſtly, 8 times 5 *l.* being 40 *l.* and the 3 carry'd making 43 *l.* the whole product appears to be 43 *l.* 6 *s.* 8 *d.*

Example of DIVISION.

l. *s.* *d.*
Divide 43 6 8 *by* 8

8) 43 6 8 (

facit 5 8 4

In dividing the above ſum, we find, firſt, that 8 will go 5 times in 43 *l.* and that 3 will remain; therefore ſetting down 5 in the place of pounds, we multiply the ſaid remainder by 20, and, taking in the 6 odd ſhillings, the produce is 66 *s.* in which again, the diviſor going 8 times, and 2 remaining, we ſet down 8 in the place of ſhillings, and multiply the 2 remaining by 12, which, with the 8 odd pence taken in, produces 32 *d.* in which, laſtly, the Diviſor 8 going exactly 4 times, 4 is ſet in the place of pence, and the whole Quotient will be found to be 5 *l.* 8 *s* 4 *d.*

Rules of PRACTICE, *with Examples.*

1. When the price is ſuch a number of pence as make an even part of a ſhilling, divide the quantity of goods by ſuch part, and the quotient will be the anſwer in ſhillings.

EXAMPLE.

Suppoſe you would know the price of 5296 ounces of Gum, at 6 *d.* per ounce ; then

oz.
6 d. being ½ } 5296
of a Shilling. }
2648 *ſhillings is the anſwer.*

For were the price 12 *d.* or 1 *s.* the number of ounces would be the number of ſhillings they would coſt : therefore, the price being 6 *d.* the ½ of a ſhilling, the anſwer muſt be but ½ ſo many ſhillings. So at 1 *d* 1 *d.* ½, 2 *d* 3 *d,* or 4 *d.* 'tis but the ½, ⅛, ⅙, ¼, or ⅓ parts of the quantity of goods, and the quotient (as aforeſaid) will be the anſwer in ſhillings, which, divided by 20, gives the pounds.

Note, If any thing remains in dividing by the ſaid parts, (pence being the next denomination) it muſt be multiply'd by 12, the pence in 1 ſhilling ; and the product divided again by the ſaid part ; and if any thing remains there, it muſt be multiply'd by 4, and ſtill divided by the ſaid part, to produce farthings.

EXAMPLE.

oz. d.
5243 at 1 $\frac{1}{2}$ *per* Ounce.

d.
1 $\frac{1}{2}$ $\frac{1}{8}$ 65|5 4 $\frac{1}{2}$

32 *l.* 15 *s.* 4 *d.* $\frac{1}{2}$ *facit.*

2. If the pence of the price are not an even part of a ſhilling, it will require more than one diviſion, to find the coſt of the goods. Thus at 5 *d.* or any other number of pence between 6 and 12.

EXAMPLES.

℔. d.
527 at 5 *per* ℔.

d.
4 $\frac{1}{3}$ 175 8
1 $\frac{1}{4}$ 43 11

21|9 7

10 *l.* 19 *s.* 7 *d.* *facit.*

℔. d.
436 at 9 *per* ℔.

d
6 $\frac{1}{2}$ 218
3 $\frac{1}{2}$ 109

32|7

16 *l.* 7 *s.* *facit.*

In the firſt of theſe, 5 *d.* is taken, thus: 4*d.* is the $\frac{1}{3}$ of a ſhilling, or the top line, and 1 *d.* the $\frac{1}{4}$ of 4 pence.

In the ſecond, the $\frac{1}{2}$ of a ſhilling is firſt taken for 6*d.* and then the $\frac{1}{2}$ of that 6 *d.* for 3 *d.*

3. If the price is both pence and farthings; for the pence, as before; and for the farthings, take the even parts of a penny: thus.

EXAMPLE.

	oz.	d.
	532	at 1 ¼ *per* Ounce.
1 *Penny*, 1½	44	4
1 *Farthing*, ¼	11	1
	5|5	5
	2 *l.* 15 *s.*	5 *d.* *facit.*

4. When the price is more than 1 *s.* and leſs than 2 *s.* leave the top line for the ſhilling, and take your parts, as before, for the remaining pence.

EXAMPLE.

d.	℔.	d.
	5437	at 19 *per* ℔.
6 ½	2718	6
1 ½ ¼	679	7 ½
	883|5	1 ½
	441 *l.* 15 *s.*	1 *d.* ½ *facit.*

5. If the price is two or more ſhillings, with pence, *&c.* either take the even parts of a pound from your top line for the ſhillings, which will give you the price in pounds; or multiply your top line by the number of ſhillings; which will give the anſwer in ſhillings, ſtill working for the pence as before. See both ways in the example.

EXAMPLE.

oz. *s.* *d.*
5263 at 2 6 *per* Ounce.
s. *d.*
2 6 $\frac{1}{8}$ 657 17 6

oz. *s.* *d.*
5263 at 2 6 *per* Ounce.
2
10526
d. 6 $\frac{1}{2}$ 2631 6
1315|7 6
657 *l.* 17 *s.* 6 *d.* *facit.*

By the firſt way, it is thus done: Had the price been 1 *l.* or 20 *s.* 'tis plain it would have come to as many pounds as there were ounces; therefore at 2 *s.* 6 *d.* which is the $\frac{1}{8}$ of a pound, it muſt come to the $\frac{1}{8}$ of ſo much money.

The other is done by ſuppoſing the top line at 1 *s.* then for 2 *s,* it muſt be twice ſo much; and for the 6 *d.* half the top line, or the value of a ſhilling, muſt be added. See both ways in another

EXAMPLE.

℔. *s.* *d.*
5276 at 19 9 *per.* ℔.
£.
10 $\frac{1}{2}$ 2638
5 $\frac{1}{2}$ 1319
4 $\frac{1}{5}$ 1055 4
6 *d.* $\frac{1}{8}$ 131 18
3 $\frac{1}{2}$ 65 19
5210 *l.* 1 *s.* *facit*

℔. *s.* *d.*
5276 at 19 9 *per* ℔.
19
47484
5276
d. 6 $\frac{1}{2}$ 2638
3 $\frac{1}{2}$ 1319
10420|1
5210 1 *s.* *facit.*

6. When the price is any thing between one and two pounds, leave the top line for the pound, and take your parts for the remainder.

EXAMPLE.

	℔.		*s.*	*d.*	
s. *d.*	5384	at	22	6	*per* ℔.
2 6 ⅛	673				
	6057 *l. facit.*				

7. When the price is two or more pounds, multiply the top line by the number of pounds; and for the remainder, take parts, as before.

EXAMPLE.

		℔.		*l.*	*s.*	*d.*	
		2364	at	4	18	10½	*per* ℔.
		4					
s.		9456					
10	½	1182					
5	½	591					
2	⅕	263	8				
1	½	118	4				
6*d*	½	59	2				
3	½	29	11				
1 ½	½	14	15 6				
		11687 *l,* 00 *s.* 6 *d.* *facit.*					

8. When with the given quantity there are odd parts, ſuch parts muſt be taken from the given price; as in the following examples.

EXAMPLES.

EXAMPLES.

		C.	qrs.	l.	s.	
		365	2 at	5	5	*per* C.
		5				
l.		1825				
5	¼	91	5			
2 *qrs.*	½	2	12	6		
		1918 *l.*	17 *s.*	6 *d.*	*facit.*	

		C.	*qrs.*	lb.	*l.*	*s.*	*d.*	
		7365	3	14 at	8	19	6	*per* C.
		8						
s.		58920						
10	½	3682	10					
5	½	1841	5					
4	⅕	1473	0					
6 *d.*	⅛	184	2	6				
2 *qrs.*	½	4	9	9				
1	½	2	4	10 ½				
14 lb	½	1	2	5 ¼				
		66108 *l.*	14 *s.*	6 *d.* ¼	*facit.*			

As in the firſt of theſe, for the 2 *qrs.* ½ the price of the C. is taken; ſo in the ſecond, not only ½ the price of the C. is taken for 2 *qrs*, but alſo ½ the price of 2 *qrs.* for 1 *qr.* and ½ that again for 14 lb. *See more examples.*

EXAMPLES.

EXAMPLES.

Tons. C. *qrs.* lb. *l. s. d.*
573 13 2 18 at 9 12 10 *per* Ton.
9

		5157		
10 *s.*	½	286	10	
2	⅕	57	06	
10 *d.*	$\frac{1}{12}$	23	17	6
10 C.	½	4	16	5
2	⅕		19	3 ¼
1	½		09	7 ½
2 *qrs.*	½		04	9 ¾
14 lb.	¼		01	2 ¼
2	⅐			2
2	⅐			2

5531 *l.* 5 *s.* 1 *d* ¼.

lb. *oz. drams.* *l. s. d.*
527 13 15 at 2 8 7 ½ *per* lb.
2

		1054		
5 *s.*	¼	135	15	
2	⅒	52	14	
1	½	26	07	
6 *d.*	½	13	03	06
1 ½	¼	3	05	10 ½
8 *oz.*	½	1	04	03 ¾
4	½		12	01 ¾
1	¼		03	00 ¼
8 *drs.*	½		01	06
4	½			09
2	¼			04 ½
1	½			02 ¼

1283 *l.* 7 *s.* 8 *d. facit.*

The

The laſt Sum another way.

lb. *oz. drams. l. s. d.*

527 13 15 at 2 8 7 ½ *per* lb.

48

4216

2108

6 *d.*	½	263	6
1 ½	¼	65	10 ½
8 *oz.*	½	24	3 ¾
4	½	12	1 ¾
1	¼	3	0 ¼
8 *dr.*	½	1	6 —
4	½		9 —
2	½		4 ½
1	½		2 ¼
		2566|7	8

1283 *l.* 7 *s.* 8 *d. facit.*

Another E X A M P L E.

lb. *Troy. oz. dwts. grs. l. s. d.*

763 11 12 13 at 23 16 4 *per* lb.

23

2208

1472

10 *s.*	½	368		
5	½	184		
1	⅕	36	16	
4 *d.*	⅓	12	5	4
6 *oz.*	½	11	18	2
4	⅓	7	18	9 ¼
1	¼	1	19	8 ¼
10 *dwts.*	½		19	10
2	⅕		3	11 ½
12 *grs.*	1			11 ¾
1	1 ½			4/4

17552 *l.* 2 *s.* 9 *d.* ½ *facit.*

Note, The foregoing examples, and all other queſtions in *Practice*, may be perform'd by the *Rule of Three*, Practice being only a compendious way of calculating ſuch propoſitions in proportion, as have an unite for their firſt number: But to ſhew the brevity of this way of working; as well as to prove the truth thereof; ſee the laſt example done by the *Rule of Three*, as follows.

By the *Rule of Three* thus:

```
   lb.         l.  s. d.               lb.  oz. dwts. grs.
If 1—coſt—23  16  4  what will   736  11   12    13?
   12          20                       12
   ----        ----                   ------
   12         476                     8843
   20          12                       20
   ----        ----                   ------
  240        5716                    176872
   24                                    24
   ----                               ------
   960                               707491
   480                              353745
   ----                             --------
  5760                              4244941
                                       5716
                                   --------
                                   25469646
                                  4244941
                                 29714587
                                21224705        12
                    576|0)24264082756|6)4212514(10
                            1224          2|0)35104|2
                            ----          -----------
                            ·720           17552 2 10¼
Facit 17552 l. 2 s. 10 d¼.  1448
                            ·2962
Three Farthings more        ·827
than by Practice loſt       ----
there, by the Fractions     2515
remaining.                  2116
                               4
                       576|0)8464(1
                             270
```

PROOF.

PROOF.

Sums in this rule may be prov'd either by taking the parts two ways, as the laſt example but one; or by the *Rule of Three*, as the laſt.

ABBREVIATIONS.

As this whole rule is indeed nothing elſe, it may be thought ſtrange to mention ſhort ways here, in a particular ſection; but as there are degrees of compariſon, ſo even ſhort ways may be more ſhorten'd.

1. The value of any quantity (the given price of one which is an even number of ſhillings) may be moſt compendiouſly found, by multiplying the ſaid quantity by $\frac{1}{2}$ their number, doubling the unites of the firſt product for ſhillings, the reſt being pounds.

EXAMPLE.

5276 *Ells of Holland*, *at* 8 *s. per Ell.*
4

2110:8 *Anſwer.*

2. If the price is an odd number of ſhillings, it is but doing as before, and adding $\frac{1}{20}$ of the top line to the product.

EXAMPLE.

Yards. *s.*
5278 at 15 *per Yard.*
7

3694:12
1 *s.* $\frac{1}{20}$ 263:18

3958 *l.* 10 *s.* *Anſwer.*

3. Were there pence in the price, their parts might alſo eaſily be taken in the ſame method.

E X A M P L E.

			lb.	*s.*	*d.*
			5384 at	7	7 ½ per lb.
			3.		
			1615	4	
1 *s.*	$\frac{1}{20}$		269	4	
6 *d.*	$\frac{1}{2}$		134	12	
1 ½	$\frac{1}{4}$		33	13	

2052 *l.* 13 *s. facit.*

4. Another ſhort and uſeful way, when the quantity is ſmall, is to multiply the price by it; thus if your quantity is expreſs'd by one figure, 'tis done by one line.

E X A M P L E.

	l.	*s.*	*d.*
5 *Yards of Velvet, at*	1	13	6 *per Yard.*
			5

8 *l.* 7 *s.* 6 *d. facit.*

5. So any quantity that can be expreſſed by 2 figures, and ſome of 3, may be eaſily and ſpeedily done thus: Find out what two figures multiply'd together, will produce the given quantity, (as ſuppoſe 32 yards or pounds, *&c.* the given quantity, which is produced by the multiplication of 8 by 4) and multiply the price by one of the ſaid figures; and the product of that, multiply'd again by the other, will give the anſwer.

EXAMPLES.

E X A M P L E S.

Yards. s. d.

32 at 15 8½ *per Yard.*

8

The value of 8 *yards* 6 5 8—

which multiply'd by 4

gives 25 *l.* 2 *s.* 8 *d. the price of* 32 *yards,*

—4 *times* 8 *being* 32.

Yards. l. s. d.

144 *at* 1 5 6 *per Yard.*

12

15 6 0

12

183 *l.* 12 *s.* 0 *d. facit.*

6. If no two figures will exactly produce thoſe given, take ſuch as will give the neareſt product to them, which will generally be within one or two, more or leſs; the value of which muſt accordingly be added or ſubſtracted.

E X A M P L E.

lb. *l. s. d.*

55 *at* 1 10 9 *per* lb.

9

The price of 9 lb. 13 16 9

which multiply by 6

gives the Price of 54 lb. 83 00 6

to which the price of 1 lb *added* 1 10 9

makes 84 11 3 *the value of* 55 lb.

So had it been multiply'd by 7 and 8, which would have given the value of 56 lb too much by 1 lb, the price of 1 lb muſt have been ſubſtracted.

7. If the quantity has odd parts, it will be eaſy taking the value of ſuch parts from the given price; as in the following

EXAMPLE.

Yds.		*l.*	*s.*	*d.*	
43 ½	*at*	2	12	7	*per Yard.*
				7	
		18	8	1	
				6	
		110	8	6	
qrs.		2	12	7	
2 ½		1	6	3 ½	
		114*l.*	7*s.*	4½	*facit.*

TRETT *and* TARE.

TARE and TRETT are allowances made to Merchants in buying their goods.

Tare, of what they can agree for *per* the whole, *per* cheſt, *&c.* or *per* C. for the weight of the bag, box, cheſt, *&c.* which contains the commodity.

Trett, for the waſte, motes, or duſt; and is always 4 lb. *per* 104 lb.

There is alſo another allowance ſometimes given of 2 lb. for every 3 C. for the turn of the ſcale, call'd cloff, or clough.

Note, The whole weight, before any allowances are made, is called *groſs*; when part is deducted, the remainder is call'd *ſuttle*; but when all are taken from it, what's left, is call'd *neat*.

RULES

RULES, *with* EXAMPLES.

1. The *Tare* of any quantity of goods may eaſily be found, if at ſo much in the whole, only by ſubſtracting the ſaid allowance from the groſs weight; if at ſo much *per* cheſt, *&c.* by multiplying the pounds tare by the number of cheſts, *&c.* and ſubſtracting as before; and if at ſo much *per* C. by taking ſuch part, or parts, of the groſs weight, as the allowance is of a C.

EXAMPLES.

Firſt ſort. Suppoſe 15 C. 2 *qrs.* 13 lb. tare were allowed on 456 C. 1 *qr.* 19 lb. of tobacco, what would be the neat weight?

	C.	*qrs.*	lb.	
From	456	1	19	*Groſs.*
Subſtract	15	2	13	*Tare.*
Remainder	440	3	6	*Neat.*

Second ſort. What's the neat weight of 3 frails of raiſins, each weighing 3 C. 2 *qrs.* 10 lb. groſs; tare at 20 lb. *per* frail?

	C.	*qrs.*	lb.		lb.
	3	2	10		20
			3 *Frails.*		3 *Frails.*
	10	3	2 *Groſs.*		60 lb. *or* 2 *qrs.* 4 lb.
		2	4 *Tare.*		
Anſwer,	10	0	26 *Neat.*		

		C.	*qrs.*	lb.		lb.	
Third ſort.		246	3	12 *Groſs.*	*Tare*	14	*per* C.
lb.							
14 $\frac{1}{8}$		30	3	12 *Tare.*			
Anſwer,		216	0	0 *Neat.*			

Again,

		C.	qrs.	lb.		lb.	
Again,		364	1	19	*Groſs.* *Tare*	16	*per* C.
	lb.						
	16 $\frac{1}{7}$	52	0	6 $\frac{1}{2}$	*Tare.*		
Anſwer,		312	1	12 $\frac{1}{2}$	*Neat.*		

Note, 14 and 16 pounds may be call'd the ſtandards of Tare; for from them may any other number of pounds, more or leſs, be taken, as in the following

EXAMPLES.

		C.	qrs.	lb.		lb.	
		573	2	13	*Groſs.* *Tare*	17	*per* C.
	lb.						
	14 $\frac{1}{8}$	71	2	22 $\frac{1}{2}$			
	2 $\frac{1}{7}$	10	0	27 —			
	1 $\frac{1}{2}$	5	0	13 $\frac{1}{2}$			
		87	0	7	*Tare.*		
Anſwer,		486	2	6	*Neat.*		

		C.	qrs.	lb.		lb.	
		548	3	11	*Groſs.* *Tare*	10	*per* C.
	lb.						
	16 $\frac{1}{7}$	78	1	17 $\frac{1}{2}$			
	8 $\frac{1}{2}$	39	0	22 $\frac{3}{4}$			
	2 $\frac{1}{4}$	9	3	5 $\frac{1}{2}$			
		49	0	0 $\frac{1}{4}$	*Tare.*		
Anſwer,		499	3	10 $\frac{3}{4}$	*Neat.*		

Note, 'Tis firſt found what tare 16 lb. would produce; becauſe 8 lb. being the $\frac{1}{14}$ part of a C. would be a troubleſome diviſor.

2. *Trett* being always 4 lb. *per* 104 lb. the conſtant method of finding it, is by taking the $\frac{1}{26}$ part of the line it is to be deducted from, 4 times 26 being 104. But it being difficult to divide by 26 in one line, it will be more ſafe and eaſy to do the work aſide, as in the firſt example.

EXAMPLES.

	C.	qrs.	lb.		lb.
	428	1	19	*Groſs.*	*Trett* 4 *per* 104
lb. 4 $\frac{1}{26}$	16	1	25 ½	*Trett.*	
Facit.	411	3	21 ½	*Neat.*	

26)428(16
168
12
4
26) 49(1
23
28
193
47
26)663(25
143
13
4
26) 52(2
0

	C.	*qrs.*	lb.		lb.
	836	2	17	*Groſs.*	*Tare* 22 *per* C.
lb.					*Tret*, 4 lb. *per* 104 lb.
16 $\frac{1}{7}$	119	2	2 $\frac{1}{4}$		
4 $\frac{1}{4}$	29	3	14 $\frac{1}{2}$		
2 $\frac{1}{2}$	14	3	21 $\frac{1}{4}$		
	164	1	10	*Tare.*	
	672	1	7	*Suttle*	
lb.					
4· $\frac{1}{26}$	25	3	12	*Trett.*	
facit	646	1	23	*Neat.*	

3. *Clough* (which is 2 lb for every 3 C.) is found, by firſt dividing the hundreds of the line it is to be taken from by 3, which brings them into 3 C'*s*; then 2 lb. being to be allow'd for every 3 C. as many 3 C's as it produces, ſo many 2 lb's it will allow; then dividing 56, (the double pounds in a hundred) the quotient will be hundreds, and the remainder double pounds; to which adding, what may be allow'd for the odd hundreds, quarters, and pounds of the given weight, it makes the whole *Clough*; which ſubtracted from the ſaid given weight, leaves the Neat.

EXAMPLES.

C.	qrs.	lb.	
5647	3	13	*Groſs.*
33	2	13 ½	*Clough.*
5614	0	27 ¾	*Neat.*

Clough 2 lb. for 3 C.

3)5647(1

	C.	qrs.	lb.
56)1882(	33	2	12
168			1 ¼
202	33	2	13 ¼
168			

14)34(2
28

6 *double Pounds.*

Note, The remainder 34 (which are double pounds) is divided by 14, (the double pounds in a *qr.*) to bring the ſaid remainder into quarters of an hundred.

Note alſo, The 1 lb. ¼ is allow'd for the odd hundred, 3 *qrs.* and 13 lb.

EXAMPLE 2.

What's the neat weight of 3 hogſheads of tobacco, weighing? *viz.*

Number	C.	*qrs.*	lb.	
1——	5	3	18	
2——	4	2	11	
3——	5	1	19	
	15	3	20	*Groſs.*
lb.				
14 $\frac{1}{8}$	1	3	27 $\frac{1}{2}$	
7 $\frac{1}{2}$		3	27 $\frac{1}{2}$	*Tare.*
	14	3	20 $\frac{1}{2}$	*Suttle.*
4 $\frac{1}{26}$		2	8 $\frac{1}{4}$	*Trett.*
	14	1	12 $\frac{1}{4}$	*Suttle.*
			9 $\frac{1}{2}$	*Clough.*
Anſwer,	14	1	2 $\frac{1}{4}$	*Neat.*

Tare 7 lb. *per* C.
Trett 4. lb. *per* 104 lb.
Clough 2 lb. *for* 3 C.

C.
3)14(4
2

lb.
Note, The 4 three hundreds allow—— 8
And the odd 2 C. 1 *qr.* 14 lb. $\frac{1}{2}$ allow—— 1 $\frac{1}{2}$
Total Clough 9 $\frac{1}{2}$

CHAP. V.

Of INTEREST.

INTEREST is either { Simple, or Compound, {

Simple, { which ariſes only from the Principal.

Compound, { which ariſes both from Principal and Intereſt.

SIMPLE INTEREST.

SIMPLE INTEREST being one of the branches of Proportion, or the *Rule of Three*, is work'd as follows.

1. The Intereſt of any Sum for one year is found by this plain proportion, *viz.*

As 100 *l.*
To the rate of Intereſt:
So is the given Principal,
To the Intereſt required.

EXAMPLE.

What's the Intereſt of 429 *l.* for a year, at 6 *l.* *per. Cent. per Annum?*

l. *l.* *l.*

If 100——6——429

6

25|74

Facit 25 *l.* 14 *s.* 9 *d.* ½. 20

14|80

12

9,60

4

2|40

Note, The cutting off two figures to the right hand, divides ſuch lines by 100. *See Abbreviations of Diviſion.*

2. To find the intereſt of any ſum for 2 or more years, multiply the year's amount by the number of years required.

EXAMPLE.

What intereſt will the foregoing principal produce in 4 years, at the ſame rate of intereſt?

l. *s.* *d.*

25 14 9 ½ *the Intereſt for one Year.*

4

Facit 102 19 2

3. To find intereſt for months, divide the year's amount by the even parts they make of a year.

EXAMPLE.

What's the intereſt of 572 *l.* for 8 months, at 5 *l.* *per Cent. per Annum?*

	l.	*l.*	*l.*		*l.*	*s.*	
If	100 ——	5 ——	572		28	12	
			5	*M.*			
				6 ½	14	6	
			28|60	2 ⅓	4	15	4
			20				
					19 *l.*	1 *s.*	4 *d. fac.*
			12|00				

4. If the time is weeks, or days, inſtead of taking the even parts of a year, as for months, it muſt be done by a ſecond ſtating in the *Rule of Three.*

EXAMPLES.

Ex. 1. What's the intereſt of 321 *l.* 16 *s.* 8 *d.* for 3 weeks, at 6 *l.* *per Cent. per Annum?*

	l.	*l.*	*l.*	*s.*	*d.*
If	100 ——	6 ——	321	36	8
					6
			19|31	00	0
			20		
Facit 19 *l.* 6 *s.* 2 *d*¼. *for a Year.*					
			6|20		
			12		
			2|40		
			4		
			1|60		

Then,

```
            Weeks.   l.  s. d.    Weeks.
Then, If 52————19  6  2¼————3
                   20
            ——————————
                  386
                   12
            ——————————
                 4634
                    4
            ——————————
                18537
                    3  4
            ————————————————
          52)55611(1069(1
             52      ————
             —   12)267(3
            ·361    ————
             312    2|2
            ————    1 l. 2 s. 3 d ¼. facit.
            ·491
             468
            ————
             ·23
```

Ex. 2. What intereſt is now due on a bond of 420 *l.* 10 *s.* commencing intereſt at 5 *l.* 10 *s.* *per Cent. per Annum*, *July* the 10th, 1727; this being *March* the 25th, 1729?

```
    l.      l.  s.       l.   s.             July——————————21
If 100————5  10————420   10                  Auguſt————————31
                           5                 September—————30
                  ——————————                 October———————31
            s.      2102  10             ————November——————30
          10 ½       210   5                 December————— 31
                  ——————————                 January ————— 31
                   23|12  15                 February ———— 28
                      20                     March ——————— 25
23 l. 2 s. 6 d ½   2|55                                 ————
  for a Year.        12                                  258
                   ——————
                   6|60
                      4
                   ——————
                   2|40
```

```
             Days.  l.  s. d.          Days.
Then, If 365—23 2 6½————258
                     20
                   ————
                    462
                     12
                   ————
                   5550
                      4
                   ————
                  22202
                    258
                  —————
                 177616
                111010
                44404
                ——————— 4
           365)5728116(15693(1
                365       ————
               ————     12)3923(11
                2078       ————
                1825     2|0)32|6
               ————        ————
                2531     16 l. 6 s. 11 d¼ for 258 days.
                2150     23 l. 2 s.  6 d½ for a-year.
               ————      ——————————
                3411     39 l. 9 s. 5 d¾ total Intereſt.
                3285
               ————
                1266
                1095
               ————
                 171
```

Another way to caſt up Intereſt, *when the principal has odd money in it, and the time, weeks or days.*

RULE.

Bring your principal into its loweſt denomination, and multiply it by the number of days; and, if at 5 *l. per Cent.* divide the product by 7300, the quotient gives the anſwer in the ſame denomination the principal was reduced into. *Note,* The number 7300 is found by multiplying 365 by 100, and dividing by 5.

EXAMPLES.

What's the Intereſt of 432 *l.* 12 *s*, 6 *d.* for 8 days, at 5 *l. per Cent. per Annum?*

```
       l.  s. d.
       432 12 6
        20
      ------
       8652
         12
      ------
      103830
           8 12
      -------
73|00)8306|40(113
      73          -----
      ---         9s. 5 d ¾. facit.
      100
       73
      ---
      276
      219
      ---
      5740
         4
      ----
73|00)229|60(3
      219
      ---
       10
```

If you would caſt up Intereſt at 6 *l. per Cent.* by this rule you muſt add ⅕ of the quotient to the intereſt produced at 5 *l. per Cent.* and ſo at other rates, it is but adding or ſubſtracting ſuch and ſuch par of the quotient to or from what it produces at ſ For inſtance, ſuppoſe the laſt queſtion done th common way, for an

EXAMPLE.

```
            l.    s.
           420   10 Principal.
             20
          ------
          8410
            623 number of days.
          ------
         25230
        16820
       50460
      ---------------s.  d.
73|00) 52394|30(717  8 3/4
       511   s.
    ---------10 1/10 71  9 1/4
       129        ------------
        73         78|9  6
      -----       ---------
       564 l.   39  9  6    Answer, 1 farthing more
       511                    than by the other way.
       -----
        5330
          12
      ----------
73|00) 639|60(8
       584
      -------
        5560
           4
      -------
73|00) 222|40(3
       219
      -------
        340
```

Note, The Rate here being 5 *l.* 10 *s.* and 10 *s.* be-ng $\frac{1}{10}$ of 5 *l.* there is therefore added $\frac{1}{10}$ of the uotient, or the amount, at 5 *l. per Cent.*

More Examples in Simple Intereſt.

At 6 *l. per Cent.* what will 589 *l.* come to in a ear?

Anſwer, 35 *l.* 6 *s.* 9 *d* ½.

What will the ſame ſum, at the ſame rate, amount to in 8 years?

Anſwer, 282 *l.* 14 *s.* 4 *d.*

What's the intereſt of 7020 *l.* for 5 months, at 5 *l. per Cent. per Annum?*

Anſwer, 146 *l.* 5 *s.*

What's a week's intereſt of 1000 *l.* at 7 *l. per Cent.*?

Anſwer, 1 *l.* 6 *s.* 11 *d.*

What will the intereſt of 5429 *l.* 10 *s.* 8 *d.* come to in 20 days, at 5 *l. per Cent.*?

Anſwer, 14 *l.* 17 *s.* 6 *d.*

What intereſt is now due on a Navy-bill, commencing intereſt at 6 *l. per Cent. per Annum, June* the 4th 1726, this being *January* the 10th 1728, the principal being 836 *l.*?

Anſwer, 8 *l.* 7 *s.* 10 *d.*

Compound INTEREST.

COMPOUND INTEREST, as was before ſaid, ariſes both from the principal and intereſt, (*i. e.*) when intereſt on money becoming due, and not paid, the ſame rate is allow'd on the unpaid intereſt, as was before on the principal.

Therefore, to work ſums in this rule, aft having found the firſt year's intereſt, as before, ad it to the principal, and find the intereſt of the ſum and ſo continue to add every year's produce, ſti accounting the ſum a new principal.

EXAMPLE.

What will be the amount of 400 *l.* forborn 3 yea and ½ at 6 *l. per Cent. per Annum*, compound intereſt

l. *l.* *l.*
If 100——6——400 *Principal* 1ſt *yea*
6

l. 24|00

l. *l.* *l.*
If 100——6——424 *Principal* 2*d. year.*
6

l. 25|44
20

s. 8|80
12

d. 9|60
4

2|40

l.
424
25 8 9 ½

l. l.
If 100 —— 6 —— 449 8 9 ½ *Princip. 3d year.*
6

l. 26|96 12 9 —
20

s. 19|32
12

d. 3|93
4

3|72

l. s. d.
449 8 9 ½
26 19 3 ¾

l. l.
If 100 —— 6 —— 476 8 1 ¼ *Princip. 4th year.*
6

l. s. d. l. 28|58 8 7 ½
28 11 8 20

M.
6 ½ 14 5 10 s. 11|68
476 8 1 ¼ 12

490 13 11 ½ d. 8|23
Total amount. 4

94

Or,
†

Or, more plain:

	l.	*s.*	*d.*
Intereſt the 1ſt Year	24	0	0
2d	25	8	9 ½
3d	26	19	3 ¾
½ Year	14	5	10
Total Intereſt	90	13	11 ¼
Principal	400	0	0
Total amount	490	13	11 ¼

More EXAMPLES.

What will 1000 *l.* amount to in 3 years, at 6 *l.* *per Cent. per Annum*, Compound Intereſt?

Anſwer, 1191 *l.* 0 *s.* 3 *d* ¾.

At 5 *l.* 10 *s.* *per Cent. per Annum*, Compound Intereſt, what will 560 *l.* 10 *s.* 9 *d.* amount to, forborn 5 years and an half?

Anſwer, 752 *l.* 14 *s.* 10 *d* ½.

CHAP. VI.

Of REBATE, *or* DISCOUNT.

REBATE, or DISCOUNT, is the abating ſo much money on a debt paid before 'tis due, as might be gain'd again by the money receiv'd, if put out to intereſt at the ſame rate, and for the ſame time. So 100 *l.* preſent money, would diſcharge a debt of 106 *l.* due at a year to come; rebate being made at 6 *l. per Cent.* becauſe 100 *l.* put out to intereſt for a year, at the ſaid rate, would regain the 6 *l.*

To work Sums in this RULE.

Firſt find the intereſt of 100 *l.* for the time; then by a *Rule of Three* ſtating, of which the firſt number muſt be 100 *l.* with the intereſt found; the ſecond number, the intereſt alone (or 100 *l.* alone, if you would find the preſent money); and the third, the debt, or ſum propounded, you will find the anſwer.

EXAMPLES.

EXAMPLES.

Ex. 1. What's the rebate of 420 *l.* for a year, at 6 *l. per Cent. per Annum?*

l. *l.* *l.*
If 106——6——420
6
l.
106)2520(23
212
·400
318
·82
20
s.
106)1640(15
106
·580
530
·50
12
d.
106)600(5
530
·70
4
106)280(2
212
·68

Answ. 23 *l.* 15 *s.* 5 *d* ½.

Ex. 2. What preſent money will diſcharge a debt of 732 *l.* due at 3 months, rebate being made at 6 *l.* *per Cent. per Annum*?

l. *l.*
6 *Intereſt for a Year of* 100
Months—
3 ¼ 1 10
l. *s.* *l.*

Then, *If* 101 10 *fall to* 100, *what will* 732
2 2
203 203)146400(721
1421
430
406
240
203
37
20
203)740(3
609
131
12
203)1572(7
1421
151
4
203)604(2
406
098

Anſw. 721 *l.* 3 *s.* 7 *d* ½.

Ex. 3. What rebate muſt be allow'd on a Bill of Exchange for 85 *l.* 10 *s.* due *September* the 8th, this being *July* the 4th, rebate being made at 5 *l.* *per Cent. per Annum*?

July	27
Aug.	31
Sept.	8
	66

```
   Days.      l.     Days.
If 365———————5———————66
                      5
                    ———
                    330
                     20
                    ———   s.  d.
           365)6600(18   0 ¾
               365
               ————
               2950
               2920
               ————
               ..30
                 12
               ————
                360
                  4
               ————
           365)1440(3
               1095
               ————
                345
```

Then,

```
           l.  s. d.        s. d.       l.  s.
Then, If  100 18 0 ¾—18 0 ¾—85 10
               20            12          20
           ---------     ---------   ---------
             2018           216        1710
               12             4          12
           ---------     ---------   ---------
            24216           867       20520
                4                         4
           ---------                 ---------
            96867                     82080
                                        867
                                     ---------
                                     574560
                                    492480
                                   656640
                                 ------------4
                        96867)71163360(734(2
                                678069   -----
                              ---------12(183(3
                                335646   -----
                                290601   15 s. 3 d½.
                              ---------
                                450450
Answer, 15 s. 3 d ½.            387468
                              ---------
                                 62982
```

Ex. 4. A Bill of Exchange for 250 *l.* is dated at *Amſterdam, June* the 14th new ſtile; at uſance is accepted, and payment offer'd the 15th *ditto* old ſtile, what muſt be then receiv'd, rebate being made at 6 *l. per Cent. per Annum?*

Note, New Stile is 11 days before Old Stile.

Uſance ſignifies a month; double Uſance, two months, *&c.* Note alſo, Merchants of *London* generally allow 3 days beyond the time appointed, calling them 3 days of grace. Therefore the bill being dated *June* the 4th *N. S.* or the 2d, O. *S.* and accepted at uſance, or a month; it becomes due *July* the 2d, O *S.* but 3 days of grace being allow'd, makes it the 6th. So if the money is paid the 15th of *June*, O. *S.* rebate muſt be made for 21 days. Therefore,

Days. *Days.* *l.*
If 365——6——21
6
126
20
s.
365)2520(6
2190
330
12
d.
365)3960(10
365
310
4
365)1240(3
1095
145

Facit 6*s*, 10*d* ¾.

If

```
   l.  s.  d.            l.          l.
If 100  6  10 ¼ ———————— 100 ——————— 250
       20                             20
   ————                          ————
   2006                          5000
     12                            12
   ————                         —————
  24082                         60000
      4                             4
  —————                        ——————    l.
  96331                  96331)24000000(249
                               192662
                               ——————
                                473380
                                385324
                                ——————
                                 880560
                                 866979
                                 ——————
                                  13581
                                     20
                                  —————— s.
                          96331)271620(2
                                192662
                                ——————
                                 78958
                                    12
                                ——————  d.
                          96331)947496)9
                                866979
                                ——————
Answer, 249 l. 2 s. 9 d ¾.       80517
                                     4
                                —————— far.
                          96331)322068(3
                                288993
                                ——————
                                 33065
```

More

More EXAMPLES.

What's the rebate of 100 *l.* for ½ a year, at 6 *l.* *per Cent. per Annum?*

Anſwer, 2 *l.* 18 *s.* 3 *d.*

What's the rebate of 538 *l.* 10 *s.* for 5 months, at 5 *l.* *per Cent. per Annum?*

Anſwer, 10 *l.* 19 *s* 9 *d* ½.

What preſent money is a debt of 629 *l.* due at 3 weeks worth, rebate beng made after the rate of 6 *l.* 10 *s.* *per Cent. per Annum?*

Anſwer, 626 *l.* 13 *s.*

What preſent money will diſcharge 10000 *l.* due at 5 days, rebate being made at 8 *l.* *per Cent. per Annum?*

Anſwer, 9989 *l.* 1 *s.* 5 *d.* ¾

CHAP. VII.

EQUATION *of* PAYMENTS.

THE Deſign of this rule is, when ſeveral ſums are due at ſeveral times, to find a mean time for paying the whole debt.

The common way to do which, is,

Multiply each ſum by its reſpective time, and add all the products together, dividing their total by the whole debt; the Quotient is accounted the mean time of paying the whole debt.

EXAMPLE.

A. owes to *B.* 40 *l.* to pay at 3 months, 60 *l.* at 5 months, and 100 *l.* at 10 months; at what time may the whole debt be paid together, without injury to either?

```
  l.       l.      l.      l.
  40       40      60     100
  60        3       5      10
 100      ———     ———    ————
————     120     300    1000
 200                     300
                         120
                        ————
                2|00) 14|20(
                        ————
```

Anſwer, 7 months.

Note,

‡

Note, The times of payment, if not ſo given, muſt be reduced into one name.

But thoſe who are more exact, take the pains to work theſe ſums by the following rule.

Firſt they find the preſent worth of every particular ſum; then find in what time to come the ſum of theſe preſent worths will be encreas'd to the total of all the particular ſums payable at the ſeveral times to come, and that is the true equated time for payment of the whole.

* * *

CHAP. VIII.

Of FELLOWSHIP, *or* COMPANY.

FELLOWSHIP, or COMPANY, is when two or more, join their ſtocks, and trade together, dividing their gain or loſs proportionably.

FELLOWSHIP is either with, or without time. Queſtions without time, are work'd by this proportion:

As the whole ſtock,
To the whole gain, or loſs:
So is each man's particular ſtock,
To his particular ſhare.

EXAMPLES.

A. B. and C. make a joint-ſtock; *A.* puts in 460 *l.* B. 510 *l.* and C. 480 *l.* they gain 340 *l.* what part of it belongs to each?

	l.
A.	460
B.	510
C.	480
Total ſtock	1450

```
                  l.        l.          l.
1st. Then, If   145|0 ——— 340 ——————— 461|0
                                        340
                                      ———————
                                       1840
                                       138
                                      ——————— l.
                                  145)15640(107
                                      145
                                      ———————
                                      ·1140
                                       1015
                                      ———————
                                       ·125
                                         20
                                      ——————— s.
                                   145)2500(17
                                       145
                                      ———————
                                       1050
                                       1015
                                      ———————
                                        ··35
                                          12
                                      ——————— d.
                                    145)420(2
                                        290
                                      ———————
                                        130
                                          4
                                      ———————
                                    145)520(3
                                        435
                                      ———————
                                        :85

     l.    l.   l.
2d. If 145|0—340—51|0
                 340
              ————————
                2040
                153
              ———————— l.
          145)17340(119
              145
              ————————
              ·284
               145
              ————————
               1390
               1305
              ————————
                 85
                 20
              ———————— s.
          145)1700(11
              145
              ————————
              ·250
              ·145
              ————————
               105
                12
              ———————— d.
          145)1260(8
              1160
              ————————
              ·100
                 4
              ————————
          145)400(2
              290
              ————————
              110
```

3d.

```
                  l.          l.        l.
3d. And, If 145|0 ——— 340 ——— 48|0
                                         340
                                       ——————
                                        1920
                                       144
                                       ——————   l.
                                    145)16320(112
                                        145
                                       ——————
                                        ·182
                                         145
                                       ——————
                                         ·370
                                          290
                                       ——————
                                          ·80
                                           20
                                       ——————   s.
                                     145)1600(11
                                         145
                                       ——————
                                         ·150
                                          145
                                       ——————
                                           ··5
                                            12
                                       ——————
                                            60
                                             4
                                       ——————
                                     145)240(1
                                         145
                                       ——————
                                         ·95
```

	l.	s.	d.		Rem.
A's *Share is*	107	17	2	$\frac{3}{4}$	— 85
B's *Share is*	119	11	8	$\frac{1}{2}$	— 110
C's *Share is*	112	11	0	$\frac{1}{4}$	— 95
Value of the Remainder				$\frac{1}{2}$	
The whole gain	340	00	0		

```
145)290(2
    290
    ———
     ·0
```

And thus are theſe Sums prov'd.

More EXAMPLES.

Three perſons make a joint-ſtock; *A* puts in 565 *l.* *B.* 278 *l.* and *C.* 629. they gain 1000 *l.* what is each of their ſhares?

		l.	*s.*	*d.*	*Remainders.*
Anſwer,	*A.*	383	16	7 ½	384
	B.	188	17	2	512
	C.	427	6	2 ¼	576

D *E.* and *F.* trade together; *D.* puts in 230 *l.* *E.* 529 *l.* and *F.* 344 *l.* 10 *s.* their whole gain amounts to 520 *l.* what is that to each?

		l.	*s.*	*d.*	
Anſwer,	*D.*	108	7	7 ¾	271
	E.	249	5	7	844
	F.	162	6	9	1092

A. *B.* and *C.* make a bank of 3256 l. whereof *A.* puts in 1026 *l.* *B* 985 *l.* and *C.* the reſt; by misfortune they loſe 2000 *l.* what part of it muſt each bear?

					Remainders.
Anſwer,	*A.*	630	4	5	928
	B.	605	0	8 ¾	1240
	C.	764	14	10	1088

FELLOWSHIP *with* TIME.

FELLOWSHIP with TIME is thus work'd: multiply each man's ſtock by the reſpective time he puts it in for, and add all the products, the total of which muſt be your firſt number through all the ſtatings; the gain, or loſs, the ſecond (as before);

fore); and each man's particular ſtock, multiply'd by its time, the third.

Note, All the particular times (if not ſo given) muſt be reduced into one name, (*i. e.*) all years, all months, all weeks, or all days, *&c.*

The reaſon of multiplying the ſeveral ſtocks by the times they are put in for, will appear from the following conſiderations. *Firſt*, ſuppoſe *A.* and *B.* put into trade each 100 *l.* for 1 month, then certainly their gain, whatever it be, muſt be equally divided betwixt them: Again, ſuppoſe *A.* puts in 100 *l.* and *B.* 200 *l.* both for the ſame time, 'tis as plain that *B*'s ſhare of the gain muſt be twice as much as *A*'s, becauſe his ſtock is double: or if each put in 100 *l.* but for different times, *viz.* *A.* for 1 month, and *B.* for 2, then as *B*'s ſtock lies twice as long as *A*'s, ſo his gain, as before, muſt be twice as great. *Laſtly*, ſuppoſe *A.* to put in 100 *l.* for 1 month, and *B.* 200 *l.* for 2 months; then ſince his ſtock is not only double, but the time he leaves it in double too, his ſhare of the profits muſt become 4 times greater than *A*'s. And ſo in all other caſes.

EXAMPLE.

A. put into company 560 *l.* for 8 months, *B.* 279 *l.* for 10 months, and *C.* 735 *l.* for 6 months; they gain'd 1000 *l.* What ſhare of it muſt each have?

	l.		*l.*		*l.*
	560		279		735
	8		10		6
A.	4480	*B.*	2790	*C.*	4410
					2790
					4480
					11680

```
                 l.        l.        l.
1st. Then, If 1168|0——1000——448|0
                             1000
                           ————— l.
                      1168)448000(383
                           3504
                           ————
                            9760
                            9344
                            ————
                            4160
                            3504
                            ————
                             656
                              20
                            ———— s.
                      1168)13120(11
                           1168
                           ————
                            1440
                            1168
                            ————
                             272
                              12
                            ———— d.
                       1168)3264(2
                            2336
                            ————
                             928
                               4
                            ————
                       1168)3712(3
                            3504
                            ————
                             208
                            l.
2d, If 1168|0 ——— 1900 — 279|0
                          .1000
                        ————————— l.
                   1168)279000(238
                        2336
                        ————
                        ·4540
                         3504
                        —————
                         10360
                          9344
                        —————
                          1016
                            20
                        ——————— s.
                   1168)20320(17
                         1168
                        ——————
                        ·8640
                         8176
                        —————
                         ·464
                           12
                        ——————— d.
                    1168)5568(4
                         4672
                        ——————
                         ·896
                            4
                        ——————
                    1168)3584(3
                         3504
                        ——————
                          ··80
```

l. *l.* *l.*

3*d*, *If* 1168|0 —— 1000 —— 441|0

1000

——— *l.*

1168)441000(377

3504

———

·5060

8176

———

·8840

8176

———

·664

20

——— *s.*

1168) 13280(11

1168

———

·1600

1168

———

·432

12

——— *d.*

1168)5184(4

4672

———

·512

4

———

1168)2048 (1

1168

———

:880

	l.	*s.*	*d.*		*Rem.*
A'*s Share is*	383	11	2 $\frac{3}{4}$	——	208
B'*s Share is*	238	17	4 $\frac{3}{4}$	——	80
C'*s Share is*	377	11	4 $\frac{1}{4}$	——	880
					——
Value of the Remainders			$\frac{1}{4}$	1168)1168(	
	1000	00	0 —		···0

The whole Gain.

More

More EXAMPLES.

A. B. and C. agreeing to trade together, *A.* puts in 592 *l.* for 4 months; B. 329 *l.* for 7 months; and C. 900 *l.* for 2 months; by their trade they gain 540 *l.* What part of it belongs to each?

		l.	*s.*	*d.*	*Remainders.*
Anſwer,	*A.*	183	14	8	2304
	B.	199	19	5	1332
	C.	156	5	10¾	2583

Three perſons make a jont-ſtock; *A.* puts in 400 pieces of holland, each containing 56 yards, at 5 *s.* 6 *d. per* ell *Flemiſh*, for 5 months; B. 600 guineas for 8 months; and C. 1800 *l.* for 4 months; but at 3 months end, takes out 800 *l.* they gain in all 1000 *l.* How much is each of their ſhares?

		l.	*s.*	*d.*	*Remainders.*
Anſwer,	*A.*	860	4	3½	3996
	B.	61	11	8	1920
	C.	78	4	0¼	2568

CHAP. IX.

Of ALLIGATION.

ALLIGATION is the compounding many ſimples into one maſs, according to any requir'd price, or proportion.

This rule is uſually divided into two parts, diſtinguiſh'd by the names of *Medial* and *Alternate*.

ALLIGATION MEDIAL.

ALligation Medial anſwers ſuch queſtions as having the quantities and prices of the ſeveral ſimples given, only demand the mean rate of any particular part of the compoſition.

RULE.

As the ſum of all the ſimples,
To their total value:
So is any quantity of the mixture,
To the price thereof.

EXAMPLES.

(*Ex.* 1*ſt.*) A Grocer would mix 18 ℔. of currants at 5 *d. per* ℔, and 29 ℔. at 6 *d. per* ℔, with 12 ℔. at 8 *d. per* ℔, At what rate may he ſell a pound of the mixture?

Firſt, place the quantities and their values, as below.

℔. *d.* *s.* *d.*
18 *at 5 per ℔. comes to* 7 6
29 *at* 6 ——— *to* 14 6
12 *at* 8 ——— *to* 8 0

Sum of all the Sim. 59 *l.* 1 10 0 *tot. val.*

℔. *l.* *s.* ℔.
Then, *If* 59 *comes to* 1 10 *what will* 1 ?
20
30
12
d.
59)360(6 *Anſwer.*
354
6

Ex. 2. A Grocer would with 12 C. of Sugar, at 3 *l. per* C. mix 3 C. at 50 *s. per* C. and 8 C. at 45 *s. per* C. What will 500 of this mixture be worth ?

The quantities and their values, placed as before, will ſtand thus :

C. *l.* *s.*
12 *at* 3 *l. p.* C. *is* 36 00
3 *at* 50 *s. p.* C. *is* 7 10
8 *to* 45 *s. p.* C. *is* 18 00

Sum of the Simples 23 61 10 *tot, val.*

Then,

C. l. s. C.

Then, *If* 23——61 10——5

20

——

1230

5

——

23)6150(26|7

46

—— 13*l.* 7*s.* 4*d* ½. *facit.*

155

138

——

170

161

——

9

12

——

23)108(4

92

——

16

4

23)64(2

46

——

18

The proof of this, is as in the *Rule of Three*, only by reverſing the proportion.

So the firſt example may be prov'd by this back ſtating.

lb. *d.* lb.

If 1——6——59

And the laſt by this,

C. *l.* *s.* *d.* C.

If 5——13 7 4 ½——23

remembring in both to take in the remainders.

More EXAMPLES.

If a vintner mingleth 13 gallons of canary, at 6 *s.* 8 *d. per* gallon, with 20 gallons of white wine, at 5 *s. per* gallon, and 10 gallons of cyder, at 3 *s. per* gallon, at what rate muſt he ſell a quart of the mixture?

Anſwer, 1 *s.* 3 *d.*

If with 40 buſhels of corn, at 4 *s. per* buſhel, there are mix'd 10 buſhels at 6 *s. per* buſhel; 30 buſhels at 5 *s per* buſhel, and 20 buſhels at 3 *s. per* buſhel, what will 10 buſhels of the miſtling be worth?

Anſwer, 2 *l.* 3 *s.*

ALLIGATION ALTERNATE.

ALligation Alternate teacheth, what quantities of ſeveral ſimples (whoſe values are given) muſt be mix'd, that the compoſition may bear a price propounded.

This ſort of *Alligation* has three varieties.

VARIETY I.

When the price of each ſimple is expreſs'd, but no quantity given, and it is required how much of each ſimple muſt be mix'd, to ſell any part of the compoſition at a mean price propounded.

In this variety you have only to link the ſeveral extreams rightly together, and to find the true differences betwixt them and the mean; for thoſe differences are the quantities ſought.

EXAMPLE.

A vintner would mix four ſorts of wine of 12 *d.* of 18 *d.* of 24 *d.* and of 26 *d. per* quart, what quantity of each muſt he take, to ſell the mixture at 20 *d. per* quart?

To place your numbers, and link the extreams, obſerve theſe rules.

R U L E S.

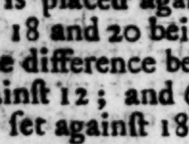
1*ſt*, Set down the ſeveral given prices (which muſt always be of one name) in a column, one under another; then, towards your left hand, draw a line of connection; and on the other ſide of the ſaid line, place the mean rate.

Thus then will ſtand the given numbers of the given example.

$$20\left\{\begin{matrix}12\\18\\24\\26\end{matrix}\right.$$

2*d*, Link them two and two together, always obſerving to join a greater and a leſs than the mean; and againſt each extream place the difference betwixt the mean and its yoke-fellow:

Thus,



So the difference betwixt the extream 12, and 20 the mean, being 8, is placed againſt 24 its yoke-fellow; and between 18 and 20 being 2, is placed againſt 26: So alſo the difference betwixt 24 and 20 being 4, is placed againſt 12; and 6, the difference betwixt 26 and 20, is ſet againſt 18.

Therefore if he mixes 4 quarters at 12 *d*. 6 at 18 *d*. 8 at 24 *d*. and 2 at 26 *d*. *per* quart, he may ſell the mixture at 20 *d*. *per* quart.

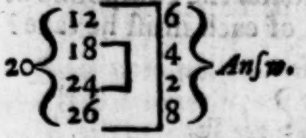
The proof of which is eaſy. For the ſum of the differences valu'd at the mean rate, will be found equal to the amount of the particular differences at their given prices.

For 20 quarts, the ſum of the differences at 20 *d. per* quart, is 400 pence; and 4, 6, 8, 2 quarts, the particular differences, at 12, 18, 24, 26 pence *per* quart will come to the ſame ſum.

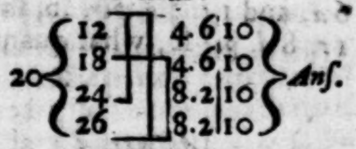
But note, as many different ways as the extreams may be combin'd, ſo many different anſwers may be given to the queſtion, yet all true:

So the ſame example being combin'd

thus:



or thus:



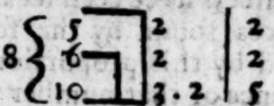
give thoſe different, but true anſwers, as may be prov'd by the ſame rule.

In the laſt of theſe examples the numbers being doubly combin'd, (*i. e.*) each with two others, make each have two differences ſet againſt it; for 12 the firſt extream, being link'd both with 24 and 26, muſt have both their differences, *viz.* 4 and 6, plac'd againſt it; and ſo the reſt. Which double difference muſt be added, and plac'd as above.

But when in a queſtion there is but one extream leſs, or but one greater than the mean; ſuch a queſtion will admit of but one anſwer: that ſingle extream being to be link'd with all the reſt.

EXAMPLE.

A grocer would mix ſugars, at 5 *d.* 6 *d.* and 10 *d. per* lb, ſo as to ſell the compound at 8 *d. per* lb; What quantity of each muſt he take?



The numbers being plac'd, link'd, and differenc'd, (as above) according to the foregoing rules, it appears to ſell his ſugar at 8 *d. per* lb, he muſt mix 2 lb. at 5 *d.* 2 lb. at 6 *d.* and 5 lb. at 10 *d. per* lb.

The reaſon of theſe combinations, and the alternate placing of their differences, will appear from this plain conſideration, *viz.* That thereby whatever is loſt upon the quantity ſold, whoſe given price exceeds the mean, is gain'd again upon the quantity, whoſe given price is leſs than the mean.

More EXAMPLES.

A tobacconiſt would mix tobacco of 2*s.* and 1 *s.* 6 *d.* and 1 *s.* 3 *d. per.* lb, ſo as to ſell the compound at 1 *s.* 8 *d. per* lb, what quantity of each muſt he take?

$$\begin{array}{r}\\ \textit{Anſwer,}\end{array}\begin{array}{c}\begin{array}{ccc}\text{lb.} & & s.\ d.\end{array}\\ \left\{\begin{array}{ccc}7 & \text{at} & 2\ 0\\ 4 & \text{at} & 1\ 6\\ 4 & \text{at} & 1\ 3\end{array}\right\}\end{array}\textit{per}\ \text{lb.}$$

Several ſorts of tea, *viz.* of 13*s.* 18*s.* 26*s.* and 30*s. per* lb, are to be ſo mix'd, that the compound may be worth 22*s. per* lb, what quantity of each muſt there be?

$$\begin{array}{r}\\ \textit{Anſwer,}\end{array}\begin{array}{c}\begin{array}{ccc}\text{lb.} & & s.\end{array}\\ \left\{\begin{array}{ccc}8 & \text{at} & 13\\ 4 & \text{at} & 18\\ 4 & \text{at} & 26\\ 9 & \text{at} & 30\end{array}\right\}\end{array}\textit{per}\ \text{lb.}$$

VARIETY 2.

Here we have the price of all the ſimples, and the quantity of one given, to find the ſeveral quantities of the reſt: ſo that one meaſure, or quantity, may be ſold at a mean rate propounded.

When the numbers have been ſet down, and their ſeveral differences found by the foregoing rules, you muſt proceed by this proportion.

As the difference of that number whoſe quantity is given,
To the reſt of the differences one after another:
So the quantity given,
To the ſeveral quantities ſought.

EXAMPLE.

EXAMPLE.

A diſtiller would with 40 gallons of *French* brandy, at 12*s*. *per* gallon, mix *Engliſh* brandy at 7*s*. *per* gallon, and ſpirits at 4*s*. *per* gallon; how much of the two laſt ſorts muſt be added, to ſell the whole at 8*s*. *per* gallon?

$$8\left\{\begin{array}{l}12\\7\\4\end{array}\right.\begin{array}{l|l}1\cdot4 & 5\\4 & 4\\4 & 4\end{array}$$

Now it is plain, that were there but 5 gallons of the *French* brandy, there muſt alſo be but 4 gallons of each of the other liquors; but ſince there are to be 40 gallons of the *French* brandy, ſay,

As 5, the difference againſt its price,
To 4, the difference againſt the price of the *Engliſh* brandy:
So is 40, the given quantity of the *French*,
To the quantity of *Engliſh* ſought,
which by this proportion will be found to be 32.
Then again,
As the firſt difference 5,
To 4, the difference againſt the price of the ſpirits;
So 40, the ſaid given quantity,
To the quantity of ſpirits ſought.
Which becauſe their differences are alike, will be the ſame as of the *Engliſh* brandy, *viz.* 32.

This variety has the ſame proof as the former.

More EXAMPLES.

How much tea at 14*s*. *per* lb, and of 18*s*. *per* lb, muſt be mix'd with 8 lb, at 24*s*. *per* lb, ſo as to allow the lb at 20*s*?

Anſwer.
8 ℔. at 24*s*.
4 ℔. at 18*s*.
4 ℔. at 14*s*.

I would with 21 lb of ſnuff, at 8*s*. *per* lb, mix ſo much of 4*s*. *per* lb, and ſo much of 5*s*. *per* lb, that I might afford to ſell the mixture at 6 *s. per* lb; what quantity of each muſt I take?

$$\textit{Anſwer},\left\{\begin{array}{l}21 \text{ at } 8\\ 14 \text{ at } 4\\ 14 \text{ at } 5\end{array}\right\}\textit{per lb.}$$

VARIETY 3.

In this variety, the particular prices of each ſimple being given, as alſo the mean rate of the compound, it is required how much of each ſort muſt be taken, to make up the quantity propounded?

Which (after the work of the firſt variety) is perform'd by this proportion:

As the ſum of the differences
To the total quantity:
So each particular difference
To its particular quantity.

EXAMPLE.

A druggiſt hath 4 ſorts of green tea, at 5 *s*. 6 *s*. 8 *s*. and 9 *s*. *per* lb, and would make up a quantity of 87 lb to ſell at 7 *s*. *per* lb, how much of each muſt he take?

See the work.

		lb.	*s.*
7	2	2—29	*at* 5 *per* lb.
	6	1—14 ½	*at* 6 *per* lb.
	8	1—14 ½	*at* 8 *per* lb.
	9	2—29	*at* 9 *per* lb.
		6	

lb. lb. lb.
If 6——87——2
2
—— lb.
6)174(29
——
0

lb. lb. lb.
If 6——87——1
1
—— lb.
6)87(14 ½
——
3

More

More EXAMPLES.

A vintner hath three ſorts of wine, of 6*s.* 8*s.* and 10*s.* *per* gallon, of all which he would compoſe a quantity of 120 gallons, to ſell at 7*s.* *per* gallon, what quantity muſt there be of each ?

	gall.		s.	
Anſwer,	80	*at*	06	*per* Gal.
	20	*at*	08	
	20	*at*	10	

A grocer having ſugars of 5*d.* 6*d.* 8*d.* 10*d.* and 11*d.* *per* lb. would make a mixture of 3 *C. wt.* to ſell at 7*d.* *per* pound, how much of each muſt he take ?

	lb.		d.	
Anſwer,	28	*at*	05	*per* lb.
	196	*at*	06	
	56	*at*	08	
	28	*at*	10	
	28	*at*	11	

CHAP. X.

Of POSITION, *or the Rule of* FALSE.

THIS rule has not the name of *Falſe* from its being in its ſelf really erroneous, but only becauſe that by the help of falſe ſuppos'd numbers, we find the truth.

It is uſually divided into two parts, *ſingle* and *double*.

Single POSITION.

IN *Single* POSITION we make but one ſuppoſition, with which working as with a true number, we regulate the reſult by this proportion, *viz.*

As the total ariſing from the error,
To the true total:
So the ſuppos'd part,
The true one.

EXAMPLES.

1ſt, *A. B.* and *C.* deſigning to buy a quantity of lead to the value of 140*l* agree that *B.* ſhall pay as much again as *A.* and *C.* as much again as *B.* what then muſt each pay?

Now, ſuppoſing *A*. to pay 10 *l*. then *B*. muſt pay 20 *l*. and *C*. 40 *l*. the total of which is 70 *l*. but ſhould be 140 *l*. therefore,

If 70 l. ſhould be 140 l. what ſhould 10 *l. be?*

l.
Anſwer, 20 *for A's ſhare,*
which doubled, makes 40 *for B's ſhare,*
and that again doubled, gives 80 *for C's ſhare.*

The total of which is 140

2. Suppoſe the ſum of 20 *l*. were to be paid by *A*. *B*. and *C*. thus, *A*. $\frac{1}{2}$, *B*. $\frac{1}{3}$, *C*. $\frac{1}{4}$, what muſt each pay according to that proportion?

	l.	*s.*	*d.*
Firſt, the $\frac{1}{2}$ *of* 20 *being*	10	00	0
the $\frac{1}{3}$ ———	6	13	4
and the $\frac{1}{4}$ ———	5	00	0
The total will be	21	13	4

but it ſhould have been but 20 *l. therefore the parts found ſhould be thus regulated:*

$$\begin{array}{l} \quad l. \;\; s. \;\; d. \qquad\qquad\quad l. \\ \textit{If } 21 \;\; 13 \;\; 4 \textit{ fall to } 20, \\ \qquad \textit{what will} \text{ ——— } \end{array} \left\{ \begin{array}{rrr} l. & s. & d. \\ 10 & 00 & 0 \\ 06 & 13 & 4 \\ 05 & 00 & 0 \end{array} \right\} \textit{Anſwer} \left\{ \begin{array}{rrrl} l. & s. & d. & \\ 9 & 04 & 7 & \frac{1}{4} \\ 6 & 03 & 0 & \frac{3}{4} \\ 4 & 12 & 3 & \frac{1}{2} \\ \hline & & & \frac{1}{2} \end{array} \right.$$

which, with the addition of $\frac{1}{2}$ *makes juſt* 20 00 0 —

$$\begin{array}{l} \textit{found by dividing the total of} \\ \qquad \textit{the remainders,} \text{ —— } \end{array} \left\{ \begin{array}{l} 28 \\ 36 \\ 40 \end{array} \right.$$

by the common diviſor 52)104(2

0

More EXAMPLES.

A gentleman bought a chaiſe, horſe, and harneſs for 50 *l*. the horſe came to twice the price of the harneſs, and the chaiſe to twice as much as both horſe and harneſs, what did he give for each ?

		l.	*s.*	*d.*		*Remainders.*
Anſwer,	*the harneſs*	05	11	1	$\frac{1}{4}$	15
	the horſe	11	02	2	$\frac{1}{2}$	30
	the chaiſe	33	06	8		

A. B. and C. diſputing of their age, *A.* affirms he is as old as *B.* and $\frac{1}{2}$ as old as C. and *B.* ſays, that he is $\frac{3}{4}$ as old as C. upon which C. ſays, he is ſure both their ages added to his, will make 110 years. I demand the age of each ?

Anſwer,	*A.*	45	$\frac{5}{6}$
	B.	27	$\frac{3}{6}$
	C.	36	$\frac{4}{6}$

Double POSITION.

IN *Double* POSITION two ſuppoſitions are requir'd, which (if both prove falſe) are, with their errors, to be thus diſpos'd.

Set down both your ſuppoſitions, and againſt each of them their reſpective errors, mark'd thus +, if too much ; or thus —, if too little.

Then multiplying them croſs ways, that is, the firſt ſuppoſition by the ſecond error, and the ſecond ſuppoſition by the firſt error ; if both the errors are alike, *i. e.* both too little, or both too much, ſubſtract the leſſer product from the greater, and divide the remainder by the difference of the errors. But if the errors are unlike, *i. e.* one too little, and the other too much, the ſum of theſe products muſt be divided by the ſum of the errors : the quotient will give the true number, exactly anſwering all the demands of the queſtion.

Or the ſuppoſitions and their errors, being plac'd as before, work by this proportion, as a general rule, *viz.*

As the difference of the errors, if alike,
Or their ſum, if unlike,
To the difference of the ſuppoſitions:
So either error to a fourth number,
which accordingly added to, or ſubſtracted from, the ſuppoſition againſt it, will anſwer the queſtion,

EXAMPLES.

1ſt, *A. B.* and *C.* playing at cards, the money ſtak'd was 324 crowns; but diſagreeing, each ſeiz'd as many as he could; *A.* got a certain number, *B.* as many as *A.* and 15 over; C. got a fifth part of both their ſums added together, how many had each?

First, *ſuppoſe A. got* 50
then B's ſhare will be 65
and C's ———— 23

All which added, make but 138 *which is* 186 *too few:*

Then, *ſuppoſing again that A. got* 80
B's ſhare will be 95
and C's ———— 35

The total of which is ſtill but 210, *too few by* 114.

Therefore, ſetting down the ſuppoſitions, and their errors (mark'd as before directed) againſt them, they will ſtand thus:

Suppoſitions.	*Errors.*
50	186
80	114

then multiply'd croſs-ways, the products will be 14880, and 5700; the leſſer of which (the errors being alike, *i. e.* both too little) being taken from the greater, there will remain 9180 for a dividend, which divided by 72, the difference of the two errors, quotes 127 ½, the true number of crowns *A.* took up; as will appear, by adding 15 to them for *B*'s ſhare, and dividing the ſum of *A*'s and *B*'s by 5 for C's ſhare; for ſo their particular numbers will be found to be as follow:

A's——127 ½
B's——142 ½
C's—— 54

The total of which is 324, the number of crowns given.

Or by the other rule,

As 72, the difference of the errors, (becauſe alike) to 30, the difference of the ſuppoſitions:

$$\textit{So is}\left\{\begin{matrix}186\\ \text{or}\\ 114\end{matrix}\right\}\textit{to}\left\{\begin{matrix}77\frac{1}{2}\\ \\ 47\frac{1}{2}\end{matrix}\right\}\begin{matrix}\textit{which}\\ \textit{added}\\ \textit{to}\end{matrix}\left\{\begin{matrix}50\\ \textit{or}\\ 80\end{matrix}\right\}\begin{matrix}\textit{gives } 127\frac{1}{2} \textit{ as}\\ \textit{before.}\end{matrix}$$

EXAMPLE 2.

When firſt the marriage-knot was ty'd
Betwixt my wife and me;
My age did hers as far exceed,
As three times three does three.
But after ten and half ten years,
We man and wife had been;
Her age came up as near to mine,
As eight is to ſixteen.
——————Now, pray,
What were our ages on the wedding-day?

First, *ſuppoſe her* 21 *then he muſt be* 63
then adding to each 15——————15

her ages becomes 36 *his* 78

This error therefore is 3 years too few.

But ſuppoſing her 9, the error will prove 3 too many.

Setting down therefore againſt the ſuppoſitions, the errors, with their reſpective marks, they will ſtand thus:

Suppo-

Suppoſitions.		*Errors.*
21	—	3
9	+	3

Then multiplying croſs-ways, 3 times 21 is 63
3 times 9 is 27
—

Theſe products (becauſe unlike) added, make 90 for a dividend, which divided by 6, the ſum of the errors, quotes 15, her true age, prov'd thus:

She being 15, *he muſt be*——45
15 *years added to each* 15
— —
Makes her 30, *juſt half of his*——60

Or, by the other rule,

As 6, the ſum of the errors, (becauſe unlike) to 12, the difference of the ſuppoſitions:

So is the common error, 3 to 6, which either ſubſtracted from 21, or added to 9, gives 15, (as before) for her age.

More EXAMPLES.

What number is that, to which if you add $\frac{1}{4}$ of its ſelf, and from the ſum ſubſtract $\frac{1}{3}$ of its ſelf, the remainder will be 210?

Anſwer, 229.

A Gentleman caught a fiſh, whoſe head was 6 inches long, the tail as long as the head, and half as long as the body; the body was juſt the length of head and tail. I demand the length of the fiſh?

Anſwer, 48 inches, or 4 foot.

CHAP. XI.

Of EXCHANGE.

EXCHANGE is the receiving money in one Country, for the value paid in another.

NOTE.

The Par of exchange is certain and fix'd, being always *par pro pari*, *like for like*, according to the weight and fineneſs of the coin.

But the Courſe or current running price, betwixt any two countries, riſes and falls upon every occaſion.

It would be both needleſs and endleſs to write of every kind of exchange; I ſhall therefore only give examples of the exchanges of *England* with ſome other chief countries of *Europe*. And firſt with

FRANCE.

At *Paris*, *Lyons*, *Roüen*, *&c.* they keep their accounts in livres, ſols, and deniers; and exchange upon the crown, the par of which in ſterling money, is 4 *s.* 6 *d.*

NOTE.

12 Deniers	make	1 Sols.
20 Sols		1 Livre.
3 Livres		1 Crown.

French money is chang'd into *Sterling* by this proportion.

As a crown to the given rate:
So is the given *French* ſum to the *Sterling* ſought.

Or,

By ſuppoſing the given *French* crowns, *goods*, and the rate of exchange, the *price*; ſo caſting up their value by *practice*.

EXAMPLES.

1*ſt*. What *ſterling* money muſt be paid in *London*, to receive in *Paris* 438 crowns; exchange at 56 *d. per* crown?

```
 Cr.      d.      Cr.             Cr.    d.    s.  d.
If 1 —— 56 —— 438              438 at 56 or 4 : 8 per Cr.
                 56
            ———————              ———————————
               2628      4 s. 1/1  87  12
               2190      8 d. 1/6  14  12
            ———————              ——————————
            12)24528(             102   4
            ———————
            2|0)204|4
            ———————
            102 l. 4 s.
```

2*d*. Change 537 crowns, 14 ſols, 10 deniers, into *Sterling* money, exchange at 52 *d* ½. *per* crown.

```
Cr.  d.       Cr. Sols De.
If 1—52 ½ —537  14  10
   60   2        60
   12 ———      ————
———105        32234
720              12
               ————
             386818
                105
          —————————
            1934090
            3868180
          ————————— 2
72|0) 4061589|0(56410
      360          ————
      ———    12) 28205(5
      461         ————
      432     2|0)235|0
      ———         ————
      295      117 l. 10s. 5d ¼. facit.
      288
      ———
       78
       72
      ———
       69
        2
      ———
  72)138(1
       72
      ———
       65
```

```
                    Cr. Sols De.   d.
                    537  14  10 at 52 ½
                    ———
       4s. 1/5      107   8
       4d. 1/12       8  19
       ½   1/8        1   2  4 ½
Sols.  10  1/6               8 ¾
        2  1/5               1 ¾
        2  1/5               1 ¾
Deniers 10 1/12 of 10 Sols.    ½
                  ——————————————
                  117 l. 10s. 5d ¼
```

Sterling money is chang'd into *French* by this proportion :

As the rate of exchange, to one crown :
So is the given *Engliſh* ſum, to the *French* ſought.

For EXAMPLE.

Take the reverſe of the former queſtions ; which will alſo be their proof.

1*ſt*. How many *French* crowns muſt be paid in *Paris*, to receive in *London* 102 *l.* 4 *s.* *Sterling* ? Exchange at 56 *d. per* crown.

```
   d.      Cr.     l.  s.
If 56———1———102  4
                 20
               ————
               2044
                 12
            ———————— Crowns.
          56)24528(438
             224
             ————
              212
              168
              ————
               448
               448
               ————
                 0
```

2*d.* Change 117 *l.* 10 *s.* 5 *d* $\frac{1}{4}$. *Sterling,* into *French* crowns. Exchange at 52 *d* $\frac{1}{2}$. *per* crown.

```
   d.          Crown.         l.   s. d.
If 52 ½ —————— 1 —————— 117  10  5 ¼
    4                        20
  ————                     ————
  210                      2350
                             12
                          —————
                          28205
                              4
       51                 ————— Crowns.   Sols.  Deniers.
         60       21|0)11282|1(537        14     10
     ———— Sols.           105
21|0)306|0(14            —————
     21                    78
    ————                   63
     96                   ————
     84                   152
    ————                  147
     12                   ————
     12                    51
  —————
    144
     66 the remainder of the other ſum.
    ————
21)210(10 deniers.
    21
   ————
    00
```

More ***EXAMPLES.***

If I pay in *Paris* 8297 *crowns*, 13 *ſols*, 8 *deniers*, what may I draw my bill for to *London*, when the exchange is at 54 *d* $\frac{1}{4}$. *per* crown?

Anſwer, 1875 *l.* 10 *s.* 4 *d* $\frac{1}{2}$.

What *Sterling* money is equal to 954 *crowns*, 52 *ſols*, when the exchange is at 58 *d* $\frac{1}{8}$. *per* crown?

Anſwer, 231 *l.* 5 *s.* 1 *d* $\frac{1}{2}$.

If I pay in *London* 536 *l.* what *French* money ſhall I receive in *Paris*, exchange at 57 *d.* *per* crown?

Anſwer, 6893 *crowns*, 50 *ſols*, 6 *deniers.*

What *French* money is equal to 1529 *l.* 10 *s.* exchange at 53 *d* $\frac{1}{4}$. *per* crown?

Anſwer, 6893 *crowns*, 31 *ſols*, 3 *deniers.*

Secondly, with ITALY.

IN *Genoa*, *Leghorn*, *&c.* they keep their accompts in livres, ſols, and deniers; and exchange upon the piece of eight, or dollar; the *par* of which with *London*, is alſo 4 *s.* 6 *d.* *Sterling.*

NOTE.

At *Genoa* 5 livres make a piece of eight, at *Leghorn* 6.

Obſerve the rules for exchanging with *France.*

EXAMPLES.

How much *Sterling* ſhall I receive in *London*, if I pay in *Genoa* 820 dollars; exchange at 51 *d.* *per* dollar, or piece of eight.

```
Dollar.      d.      Dollars.   Dollars.
If 1 ————— 51 ————— 820         820 at 51 d. per dollar.
                      51           4
                    ————        ————
                  820 d.        3280
                 4100     3 1/4  205
                 ————         ————
             12)41820(         348|5
                 ————         ————
             2|0)348|5        174 l. 5 s.
                 ————
                174 l. 5 s.
```

Or, back again;

If in *London* I pay 174 *l.* 5 *s.* Sterling, how many dollars ſhall I receive in *Genoa?* exchange at 51 *d.* *per* dollar.

d. *Dollar.* *l.* *s.*

If 51——1——174 5

20

3485

12

51) 41820(820

408

102

102

...0

Venice exchanges by the ducat, the *par* about 56 *d* ¼. *Sterling.*

NOTE.

6 Solidi } make { 1 Groſs

24 Groſſes } make { 1 Ducat.

EXAMPLE.

Change 512 ducats into *Sterling*-money, exchange at 55 *d* ½. *per* ducat.

D. *d.* *D.*

If 1—55 ½ —512 *

2 512

512

111

2) 56832(

12)28416(

2|0) 236|8

118 *l.* 8 *s.*

d.

6 ½

1 ½ ¼

Ducats. *d.*

512 at 55 ½ *per* Duc.

4

2048

256

64

236|8

118 *l.* 8 *s.*

---

* *See* Abbreviations of Multiplication.

Thirdly, with SPAIN.

IN *Madrid*, *Sevil*, &c. they keep their accompts in rials and mervadies, and exchange alſo by the piece of eight. The *par* with *London* 4*s*. 6*d*.

NOTE.

372 Mervadies	make	1 Rial.
8 Rials		1 Piece of $\frac{8}{8}$

This exchange is alſo caſt up by the foregoing rules.

EXAMPLE.

What *Sterling* money may I draw for to *London*, if I pay in *Sevil* 4798 rials, 250 mervadies? exchange at 58 *d*. *Sterling per* piece of $\frac{8}{8}$.

```
Rials.   d.    Rials. Mer.
If 8——58——4798 250
  372          372
  ————       ————
  2976         9596
              33591
             14396
             —————
             1785106
                  58
             ————————
            14280848
            8925530
            ————————  12
2976)103536148(34790(2
       8928         ————
       —————2|0)289|9(
       14256      ————
       11904      144 19 2 ¼
       —————
        23521
        20832
        —————
        26894
        26784
        —————
         1108
            4
        —————
   2976)4432(1
        2976
        ————
        1456
```

Or, back again,

Change 144 *l.* 19 *s.* 2 *d* ¼. *Sterling* into *Spaniſh* money. Exchange at 58 *d.* *per* piece of $\frac{8}{8}$.

d. *Rials.* *l.* *s.* *d.*
If 58——8——144 19 2¼
4 20
232 2899
12
34790
4
139161
8
232)1113288(4798 *Rials.*
928
1852
1624
2288
2088
2008
1856
152
372
304
1064
456
1456 *Rem. of the other.*
232)58000(250 *Mervadies.*
464
1160
1160
...0

More EXAMPLES.

Change 83927 *Rials* into pounds *Sterlings*, exchange at 54 *d* ½ *per* piece of eight.

Anſwer, 2382 *l.* 6 *s.* 0 *d* ½.

If I pay in *London* 5000 *l.* what *Spaniſh* money may I draw my bill for to *Madrid?* exchange at 57 *d.* ⅛ *per* piece of eight.

Anſwer, 168052 *Rials*, 192 *Mervadies.*

Fourthly, with PORTUGAL.

IN *Lisbon*, *Oporto*, &c. they keep their accompts in Rees, and exchange on the Mill-ree; the *par* of which is about 6 *s.* 8 *d* ½. *Sterling.*

NOTE.

1000 Rees make 1 Mill-Ree.

To make this exchange, ſtill obſerve the ſame rule.

EXAMPLES.

Change 48009 rees into Sterling money, exchange at 6 *s.* 5 *d. per* mill-ree.

```
M. R.    s. d.       Rees.
If 1 ——— 6  5 ——— 48009
1000    12            77
        —          ——————
        77        336063
                 336063
                ———————— 12
1|000)3696|693(3696(
           4        ————
      ————————   2|0)30|8
         2|772    ————————
                  15 l. 8 s. 0 d ½.
```

Or, back again;

Change 15 *l.* 8 *s.* 0 *d* ½. *Sterling* into rees, exchange at 6 *s.* 5 *d.* *per* mill-ree.

Rees. *l.* *s.* *d.*

If 6 *s.* 5 *d.*——1000——15 8 0 ½

12 — 20

77 — 308

4 — 12

308 — 396

4

14786

1000

Rees.

Note, 772, *Rem. of* 308) 14786772(48009

the other Stating, is here taken in.

1232

2466

2464

2772

2772

0..

More EXAMPLES.

What *Sterling* money is equal to 595677 rees-exchange, at 6 *s.* 7 *d.* *per* mill-ree?

Anſwer, 196 *l.* 1 *s.* 6 *d* ¼.

Change 59 *l.* 12 *s.* into rees, exchange at 6 *s.* 10 *d* ½. *per* mill-ree?

Anſwer, 173381 rees.

Fifthly,

Fifthly, with HOLLAND, FLANDERS, *and* GERMANY.

I Take theſe places together, becauſe their accompts are kept, and their exchange with *England* made the ſame way.

For not only in *Amſterdam*, but alſo in *Antwerp* and *Hamborough*, they keep their accompts in pounds, ſhillings, and pence *Flemiſh*; or guilders, ſtivers, and pennicks; and exchange with us upon our pound, giving us for it, when at *par*, 33 *s.* 4 *d.* *Flemiſh*.

NOTE.

Their pounds, ſhillings, and pence, are divided as ours, *viz.* their pound into 20 ſhillings, and their ſhilling into 12 pence.

Their other money is thus divided.

16 Pennicks } make { 1 Stiver.
20 Stivers } make { 1 Guilder.

Note alſo,

6 Stivers } make { 1 Shilling } *Flemiſh*.
6 Guilders } make { 1 Pound } *Flemiſh*.

Sterling money is chang'd into *Flemiſh* by this proportion, *viz.*

As 1 *l.* *Sterling* to the given rate:
So is the given *Sterling* to the *Flemiſh* ſought.

Or,

By ſuppoſing the given Sterling, *goods*, and the given rate, their *price*, caſting up their value by *practice*.

EXAMPLE.

EXAMPLE.

If I pay in *London* 492 *l.* Sterling, what may I draw my bill for to *Amſterdam*? exchange at 34 *s.* 5 *d. Flemiſh per* pound Sterling.

l. — *s.* *d.* — *l.*

If 1 —— 34 5 —— 492

12 — 413

Or by Practice. 413 — 1476

l. *s.* *d.* — 492

492 at 34 5 *per l.* — 1968

s. 10 $\frac{1}{2}$ 246

4 $\frac{1}{5}$ 98 8 — 12)203196(

d. 4 $\frac{1}{12}$ 8 4

1 $\frac{1}{4}$ 2 1 — 2|0)1693|3(

846 *l.* 13 *s. facit.* — 846 *l.* 13 *s. Flemiſh.*

If you would have the anſwer in guilders and ſtivers,

Divide the *Flemiſh* pence by 40, (the number of pence which make a guilder) and the quotient will be guilders; and if any thing remains, the half of it will be ſtivers, two *Flemiſh* pence being one ſtiver.

Or,

Multiply the *Flemiſh* pounds, and ſhillings, by 6, and it will produce guilders and ſtivers; for 6 guilders make 1 pound *Flemiſh*, and 6 ſtivers 1 ſhilling *Flemiſh*, and if there be any odd pence, multiply them by 8, for *Pennicks*.

EXAMPLE.

846 *l.* 13 *s.* *the facit of the laſt ſum.*

6 — *Flemiſh pence.*

4|0)20319|6(36 *Rem.*

5079 G. 18 *S.*

5079 *Guild.* 18 *Stivers.*

Flemiſh money (whether pounds, ſhillings, *&c.* or guilders, ſtivers, and pennicks) is exchanged into Sterling thus:

As the given rate to 1 pound Sterling: ſo is the given *Flemiſh* to the Sterling ſought.

EXAMPLES.

Change 846 *l.* 13 *s. Flemiſh* into Sterling money, exchange at 34 *s.* 5 *d. Flemiſh per l.* Sterling.

```
    s.  d. Flem. l.   Sterl. l.  s. Flem.
 If 34  5————————1————————846 13
    12                     20
   ————                  ——————
    413                   16933
                             12
                         ——————
                   413)203196(492 l. Sterling.
                       1652
                       ——————
                        3799
                        3717
                       ——————
                          826
                          826
                         ————
                            0
```

Or,

Change 5079 guilders, 18 ſtivers into Sterling money, exchange 34 *s.* 5 *d. per l.* Sterling.

If

```
   s.  d. Flem.   l. Ster.   Guild. Stiv.
If 34  5 ———————— 1 ———————— 5079  18
   12                          20
   ——                        ——————
   413                       101598
                                  2
                             ——————
                       413) 203196(492 l. Sterl.
                            1652
                            ————
                            3799
                            3717
                            ————
                             826
                             826
                             ———
                               0
```

More EXAMPLES.

Change 475 *l.* Sterling into *Flemiſh*-money; exchange at 33 *s. Flemiſh per l.* Sterling?

Anſwer, 783 *l.* 15 *s. Flemiſh.*

What *Flemiſh* money is equal to 365 *l.* 12 *s.* exchange at 36 *s.* 5 *d. per l.* Sterling?

Anſwer, 665 *l.* 13 *s.* 11 *d. Flemiſh.*

If I pay at *Amſterdam* 5389 *l.* 14 *s. Flemiſh*, what Sterling money ſhall I receive at *London*, exchange at 32 *s.* 8 *d. Flemiſh per l.* Sterling?

Anſwer, 3299 *l.* 16 *s.* 3 *d* $\frac{1}{4}$.

Change 572 *l.* 13 *s.* Sterling into guilders, ſtivers, *&c.* Exchange at 34 *s.* 10 *d. Flemiſh per l.* Sterling.

Anſwer. 5984 *guilders*, 3 *ſtivers*, 13 *pennicks.*

What may I draw my bill for to *London*, if I pay in *Antwerp* 7256 *guilders* exchange, at 35 *s.* 2 *d* $\frac{1}{2}$. *per l.* Sterling?

Anſwer, 686 *l.* 19 *s.* 2 *d.* Sterling.

An

An Explanation of ſome Characters ſometimes us'd for Brevity's Sake.

$+$ IS the mark of *Addition*, and ſhews the numbers it's plac'd between, are to be added together.

$-$ Is the mark of *Subſtraction*, and ſhews the latter of the numbers it ſtands between, is to be taken from the former.

$\times$ Is the mark of *Multiplication*, and ſhews the numbers on each ſide, are to be multiply'd together.

$\div$ Is the mark of *Diviſion*, and plac'd between two numbers, ſhews the firſt is to be divided by the laſt.

$=$ Is the mark of Equality, and ſhews the number or numbers preceding it, are equal to that which follows it.

$\surd$ Plac'd before any figure, ſhews the Square Root of them is meant. And

$\surd c$ before a number, denotes its Cube-Root.

$::$ Is the ſign of *Proportion*, and being plac'd betwixt the two middle terms of 4 proportionals, ſignifies, *ſo is*.

To make all more plain, obſerve,

Thus the Characters are us'd:

$$2:4::6:12\quad \surd c\,4096=16,\ \surd 16=4,\ 4+2=6,$$
$$6-2=4,\ 4\times 2=8,\ 8\div 2=4.$$

And thus read:

As 2 is to 4, ſo is 6 to 12; the Cube-Root of 4096 is, or is equal to, 16; the Square Root of 16, is equal to 4; 4 added to, or more by 2, is equal to 6; and 6, made leſs by 2 is equal to 4; again, 4, multiply'd by 2, is equal to 8, and 8 divided by 2, is equal to 4.

CHAP.

CHAP. XII.

Of *Arithmetical and Geometrical* PROPORTION, *or* PROGRESSION.

PROGESSION being a rule more curious than uſeful, and requiring more words for its full explication, than are conſiſtent with this *Compendium,* I ſhall only give ſome (the moſt neceſſary) propoſitions in both its parts, arithmetical and geometrical; and refer the more inquiſitive reader to Mr. *Oughtred*'s *Clavis Mathematica,* for his further ſatisfaction.

In all continued proportion, or compariſon of numbers, that number which is compared to another is called the *antecedent,* and the number to which it is compared is called the *conſequent.*

There are two ways of comparing numbers with one another. 1*ſt.* With reſpect only to their ſimple difference, *i. e.* how much the one, *viz.* the *antecedent,* is greater, or leſs than the other, *viz.* the *conſequent,* found by ſubſtraction; thus the difference betwixt 5 and 8, is 3; which difference is called the *Arithmetical Ratio.* 2*d.* Numbers are compared with one another, when we conſider how often one number contains, or is contained in another; and this is found by diviſion, the quotient being the *geometrical ratio.* Thus, 2 is the *geometrical ratio* betwixt 8 and 16.

ARITHME-

ARITHMETICAL PROGRESSION.

ARITHMETICAL PROGRESSION is the regular increaſe or decreaſe of any rank of numbers, by the continual addition or ſubſtraction of ſome equal number.

So 1, 4, 7, 10, 13, 16, are in *Arithmetical Progreſſion* to each other, equally increaſing by the continual addition of 3.

And ſo alſo are 84, 82, 80, 78, regularly decreaſing by the continual ſubſtraction of 2.

Some of the moſt conſiderable properties of numbers in *Arithmetial Progreſſion* are as follow:

1. That in any rank of numbers in *Arithmetical Progreſſion* conſiſting of three, or any odd number of terms, the double of the middle term will be equal to the ſum of the two extreams, or of any two means equally diſtant from the ſaid middle term.

1, 2. 3, 4. (5) 6, 7, 8, 9.

Thus in the rank above 5, the middle term doubled is 10, = to 1 + 9, the two extreams, or 2 + 8, or 3 + 7. or 4 + 6, the ſeveral correſpondent means.

2. That in any rank of numbers in *Arithmetical Progreſſion* conſiſting of four, or any even number of terms, the ſum of the two extreams will be equal to the ſum of the two middle numbers, or of any two means equally diſtant from the ſaid extreams.

2, 4, 6, 8, 10, 12, 14, 16.

Thus in the rank above 2 + 16, the two extreams are 18, = 8 + 10; the two middle numbers, or 6 + 12, or 4 + 14, the other correſpondent means.

In PROGRESSION, *five Things are to be noted,*

VIZ.

I. The firſt Term.
II. The laſt Term.
III. The number of Terms.
IV. The equal Difference.
V. The Sum of all the Terms.

Any three of which being given, the other two may be found; as Mr. *Oughtred* in his *Clavis Mamaticæ*, *Chap.* 19. *Prob.* the 6th, exemplifies in 20 propoſitions; ſome of the moſt uſeful of which are as follow.

PROPOSITION I.

The laſt term, number of terms, and equal difference being given, to find the firſt term:

Or the ſecond, third, and fourth given, to find the firſt.

RULE.

Multiply the fourth by the third, made leſs by one, and ſubſtract the product from the ſecond, the remainder is the anſwer.

EXAMPLE.

A man takes out of his pocket, at 6 ſeveral times, 6 ſeveral numbers of ſhillings, every one exceeding the former by 6, the laſt was 42, what was the firſt? *Anſwer*, 12*s.*

$6 \times 5 = 30$, $42 - 30 = 12$ *Anſwer.*

PROPOSITION II.

The firſt, third, and fourth given, to find the ſecond.

RULE.

From the product of the fourth, multiply'd by the third, ſubſtract the fourth, the remainder added to the firſt, gives the ſecond.

EXAMPLE.

What's the laſt number of an *Arithmetical Progreſſion*, beginning at 4, and continuing, by the increaſe of 12 to 18 places? *Anſwer*, 208.

$12\times18=216$, $216-12=204$, $204+4=208$ *Anſwer.*

PROPOSITION III.

The firſt, ſecond, and fourth given, to find the third.

RULE.

Subſtract the firſt from the ſecond, and divide the remainder by the fourth, the quotient, with one added, gives the third.

EXAMPLE.

A traveller went 12 miles the firſt day, and increas'd every day's journey by 4 miles, till at laſt he went 64 miles in one day; How many days did he travel? *Anſwer*, 14 days.

$64-12=52$, $52\div4=13$, $13+1=14$ *Anſwer.*

PROPOSITION IV.

The firſt, ſecond, and third given, to find the fourth.

RULE.

Subſtract the firſt from the ſecond, and divide the remainder by the third, made leſs by one, the quotient gives the fourth.

EXAMPLE.

The ages of ſeven children increaſe by *Arithmetical Progreſſion*; the youngeſt is two years old, the eldeſt 32, what's the common difference of their ages? *Anſwer*, 5 years.

32—2=30, 30÷7—1=5 *Anſwer.*

PROPOSITION V.

The firſt, ſecond, and third given, to find the fifth.

RULE.

Half the ſum of the firſt and ſecond, multiply'd by the third, produces the fifth.

EXAMPLE.

12 perſons give their charity to a poor man in *Arithmetical Progreſſion*; the firſt gave 2 pence, the laſt 2 ſhillings, or 24 pence; how much did the poor man get? *Anſwer*, 156 pence, or 13 ſhillings.

2+24=26, 26÷2=13, 13×12=156 *Anſwer.*

GEOMETRICAL PROGRESSION.

GEOMETRICAL PROGRESSION is the increaſing or decreaſing of any rank of numbers by ſome equal *Ratio*, that is, by the continual multiplication or diviſion of ſome equal number.

Thus, 2, 4, 8, 16, 32, increaſe in *Geometrical Progreſſion* by a double *Ratio*, or continual Multiplication by 2, and 405, 135, 45, 15, 5, decreaſe after the ſame manner by a continual Diviſion by 3, or by a triple *Ratio*.

Some of the moſt conſiderable properties of numbers in *Geometrical Proportion*, are as follow:

1. That in any rank of numbers in *Geometrical Progreſſion*, conſiſting of three, or any odd number of terms, the product of the middle term multiplied by its ſelf, will be equal to the product of the two extreams, or of any two means equally diſtant from the ſaid middle term.

1, 2, 4, (8) 16, 32, 64.

Thus in the rank above, 8, the middle term ſquar'd, *i. e.* multiply'd by its ſelf, is 64, = 1 × 64, the two extreams, or 2 × 32, or 4 × 16, the correſpondent means.

2. That in any rank of numbers in *Geometrical Progreſſion*, conſiſting of four, or any even number of terms, the product of the two extreams, wil be equal to the product of the two middle numbers, or of any two means equally diſtant from the ſaid extreams.

2, 4, 8, 16, 32, 64.

Thus in the rank above 2 × 64, the two extreams produces 128 = 8 × 16, the two middle numbers, or to 4 × 32, the other correſpondent means.

In Geometrical Progreſſion, *the ſame five things are to be noted as in* Arithmetical Progreſſion.

V I Z.

I. The firſt Term.
II. The laſt Term.
III. The number of Terms.
IV. The *Ratio*, or equal difference.
V. The Sum of all the Terms.

Note, Figures are ſometimes ſet over a rank of geometrical proportionals as hereunder, and are called *Indices*, or *Exponents*, ſerving to ſhew the diſtance of any term from unity, or from the firſt term of the ſeries.

Thus 0 1 2 3 4 5 6
1 . 2 . 4 . 8 . 16 . 32 . 64

PROPOSITION I.

To find any remote term of a *Geometrical Progreſſion* proceeding from unity, whoſe *ratio* or common difference is known, without producing all the intermediate terms.

RULE.

Find ſome few of the leading terms, and place over them their Exponents; then multiplying the laſt found term by its ſelf, it will produce a term double thereto; which again multiply'd by its ſelf, will produce another double to the laſt: Thus proceed, till either you produce the term ſought, or one a little ſhort of it; which may be compleated by multiplying it again by that term, which ſtands under ſuch exponent as would make up the number.

Or in other words, obſerve what figures of the exponents found added together, would give the exponent of the term wanted, and the numbers ſtanding under the ſaid exponents, multiply'd into each other, will produce the term requir'd.

From whence may be obſerv'd the coherence or agreement betwixt numbers in arithmetical and geometrical proportion.

EXAMPLE.

One agrees, for 14 oranges to pay only the price of the laſt, at a farthing for the firſt, an half-penny the ſecond, *&c.* ſtill doubling the price for the next, what muſt he give?

Here ſetting down only the five firſt terms, wit their exponents, the fourteenth may thus be found 5 + 5 + 4 = 14. So the terms under thoſe exponents being multiply'd into each other, give th term anſwering. But as the exponent 1 ſtands always over the ſecond term, ſo the exponent 13 anſwers to the 14 term; therefore add only 3 to th two 5's; and ſo always that exponent muſt b taken, which is one ſhort of the term requir'd, as for 20, 19; for 30, 29, *&c.* which muſt be carefully noted.

See the work of the foregoing example.

0 . 1 . 2 . 3 . 4 . 5
1 . 2 . 4 . 8 . 16 . 32

Exponents.	*Terms.*	
5	32	*the 5th multiply'*
5	32	*by its ſelf.*
3	64	
13	96	
	gives 1024	*the 10th, which*
	multiply'd by——8	*the 3d Term,*
	gives 8192	*the 14th.*

And ſo many farthings, or 8 *l.* 10 *s.* 8 *d.* muſt he give at the rate agreed.

PROPOSITION II.

To find any remote term of a *Geometrical Progreſſion,* not proceeding from unity, whoſe *ratio* and firſt term is given, without producing all the intermediate terms.

RULE.

RULE.

Set down, as before, ſome few of the leading terms with their exponents, and multiply alſo, (as in the firſt propoſition) the terms under ſuch exponents, as added, would make the exponent ſhort by one of the terms requir'd; only remember that every product muſt here be divided by the firſt term.

EXAMPLE.

A ſum of money is thus to be divided among 9 perſons, the 1ſt to have 50 *l.* the 2d 150 *l.* and ſo on in that proportion, one three times more than the other, to the laſt; what will the laſt have?

Setting down the five firſt terms with their exponents, thus,

```
 0   1    2    3    4
50  150  450  1350  4050
```

Multiply the laſt term by its ſelf, and divide the product by 50, the firſt term, the quotient will be 328050 the 9th term requir'd, anſwering to the 8th exponent.

Or ſetting down but the three firſt terms, multiply the laſt by its ſelf, and divide the product by the 1ſt term, it quotes the 5th: which again multiply'd by its ſelf, and divided by the 1ſt, will quote the 9th, as before. See the work.

```
 0   1   2       450              4050
50  150 450      450              4050
             ---------         ----------
               22500             202500
              1800             162000
             ---------         ----------
      5|0)2025 0|0(       5|0)1640250 0|0(
             ---------         ----------
               4050              328050
```

Anſwer, 328050 *l.* for the 9th perſon.

PROPOSITION. III.

Having the firſt term, the common exceſs, and the number of places given, to find the total ſum of the whole progreſſion.

RULE.

Having found the laſt term by the foregoing rules, ſubſtract from it the firſt, and divide the remainder by the common exceſs, made leſs by one; the quotient is the ſum of all, except the laſt, to which, that being alſo added, gives the total required.

Note, If the *ratio* be double, that is, if the common exceſs be 2, the difference between the leaſt and the greateſt term added to the greateſt, gives the total ſum. If the *ratio* be triple, ½ the difference muſt be added; if quadruple, ⅓, *&c.*

EXAMPLES.

1ſt, A coffee-man, upon the ſigning of the laſt Peace, for 50 guineas down, agreed to pay the firſt day one coffee-berry, the ſecond two, the third four, and ſo on, to double the quantity every day, till the ſame was proclaim'd. What number of berries would it amount to, ſuppoſing the time 60 days; and what would their value be, ſuppoſing 1000 berries to the pound, and the pound to be ſold at 5*s*?

See the work.

Days.	*Berries.*
1	1
2	2
3	4
4	8
5	16

16

16

96

16

256

2

512 *the* 10*th day.*

16

3072

512

8192

2

16384 *the* 15*th day.*

16384

65536

131072

49152

98304

16384

268435456

2

536879012 *the* 30*th day.*

536870912

1073741824

536870912

4831838208

37580963840

4294967296

3221225472

1610612736

2684354560

2882303761517111744

2

5764607523034223488 *the* 6*th day.*

1|000(1152921504606846|975 *Tot. number of Berries.*

5 *s.* $\frac{1}{4}$ 288230376151711 *l.* 10 *s. their val. at* 5 *s. per* ℔.

Ex. 2. A grazier offers 40 oxen for a farthing a head, and treble it throughout; to what ſum will it amount?

See the work.

```
Oxen. Farthings.
   1———— 1
   2———— 3
   3———— 9
   4————27
   5————81
        81
     —————
        81
      648
     —————
      6561
         3
     —————
     19683 10th Ox.
     19683
     —————
     59049
   157464
  118098
 177147
 19683
 ——————————
 387420489
         3
 ——————————
1162261467 the 20th Ox.
```

```
            1162261467
            1162261467
            ——————————
            8135830269
           6973568802
          4649045868
         1162261467
        6973568802
       2324522934
      2324522934
     6973568802
    1162261467
   1162261467
   ———————————————————
   1350851717672992089
                     3
   ———————————————————
   4052555153018976267 40th Ox.
   ———————————————————
   6078832729528464400 Tot. Far.
           Or,
  6332117426592150 l. 8 s. 4 d.
```

CHAP. XIII.

Of VULGAR FRACTIONS.

A FRACTION is a part or parts of an unit, and written with two figures, with a line drawn between them, as $\frac{1}{4}$, $\frac{2}{3}$, $\frac{6}{8}$.

The figure under the line is call'd the ***denominator***, becauſe it gives name to the fraction; and alſo ſhews into what parts the unit is broken; and that above the ***numerator***, becauſe it tells how many of ſuch parts are meant by the fraction. Thus the fraction $\frac{3}{4}$ ſhews, that the unit is divided into 4 parts, and that 3 of thoſe parts are thereby expreſs'd.

1. *Note* therefore, That if the ***numerator*** and ***denominator*** of a fraction ſhould be equal as $\frac{4}{4}$, $\frac{6}{6}$, the value of ſuch fraction would be exactly an unit or integer; for, by the definition above, the ***denominator*** ſhews into how many parts the unit is broke; and the ***numerator*** expreſſes how many of thoſe parts are meant by the fraction. Then in the fraction $\frac{4}{4}$, the ***denominator*** declaring the unit to be broke into 4 parts, and the ***numerator*** expreſſing 4, that is, all of thoſe parts, it is plain the ſaid fraction is equal to an unit or whole number; becauſe the ſum of all the parts muſt muſt be equal to the whole. From which obſervation it is alſo plain, that ſo often as the ***denominator*** is contain'd in the ***numerator***, ſo many whole numbers or units are contain'd in ſuch im-

proper fraction. And this *Note* may ſerve as a reaſon for the operations of the 1ſt, 2d, and 3d ſorts of Reduction.

2. *Note* alſo, That as all fractions do indeed ariſe from the remainders of diviſions, when the diviſor can no longer meaſure the dividend, ſo every fraction may be look'd upon as the two given terms of a diviſion, the *numerator* as the dividend, and the *denominator* as the diviſor; from whence it appears, that if the *numerator* and *denominator* of a fraction be either multiply'd or divided both by the ſame number, the products or quotients will ſtill remain in the ſame proportion, and the new fraction ſo ariſing, be of the ſame value with that given. Thus the fraction $\frac{2}{4}$, multiply'd by 2, will produce $\frac{4}{8}$, or divided by 2, will quote $\frac{1}{2}$; all which fractions are of the ſame value; 4 bearing the ſame proportion to 8, and 1 to 2, as 2 does to 4. And from hence alſo may appear the reaſon of the 1ſt, 5th, and 6th ſorts of Reduction.

Of Vulgar Fractions, *there are four ſorts*, viz.

1ſt. A *proper*; whoſe *numerator* is always leſs than its *denominator*, as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{6}$, *&c.*

2d. An *improper*; whoſe *numerator* is always equal to, or greater than its *denominator*, as $\frac{5}{3}$, $\frac{6}{4}$, *&c.*

3d. A *compound*, or *fraction of a fraction*; known by the word *of*, as $\frac{3}{4}$ *of* $\frac{5}{6}$, *&c.*

4th. A *mix'd fraction*; which is a fraction join'd with a whole number, as $5\frac{1}{2}$, $3\frac{2}{3}$, *&c.*

Before fractions can be either added or ſubſtracted, they muſt be reduced into one denomination; and therefore, before we proceed to thoſe rules, we muſt learn reduction.

REDUCION *of* FRACTIONS.

IN *reduction of fractions* there are ſeven varieties.

Firſt, To reduce a whole number into an *improper fraction*, which is done by placing 1 for its *denominator*.

EXAMPLES.

Reduce 4 to a Fraction, *facit* $\frac{4}{1}$
or 18, ——— *facit* $\frac{18}{1}$.

But if you would aſſign it any other *denominator*, multiply the whole number by the *denominator* aſſign'd, and place the product for a *numerator* over the aſſign'd *denominator*.

EXAMPLES.

Reduce 12 to a fraction, whoſe *denominator* let be 8. $9\frac{8}{8}$ *facit* $\frac{96}{8}$.

Or if 6 were to be made a fraction, and its *denominator* to be 7, it would become $\frac{42}{7}$.

For the reaſon of this rule conſider the two foregoing notes.

Second, To reduce a mix'd fraction into an improper one,

Multiply the whole number by the *denominator* of the fraction given, and take in the *numerator*; the product place for a new *numerator* over the *denominator* given.

EXAMPLES.

Reduce $5\frac{1}{3}$ to an improper fraction, *facit* $\frac{16}{3}$

$$\begin{array}{c} 3 \\ \hline 16 \\ \hline 3 \end{array}$$

So $12\frac{1}{2}$ reduc'd to an improper fraction will be $\frac{25}{2}$.

The reaſon of this rule is the ſame as the foregoing, there being no difference in the operation, but the taking in the given *numerator*.

Third, To reduce an improper fraction into its equivalent whole or mix'd number.

RULE.

Divide the *numerator* by the *denominator*, the quotient gives the whole number contain'd: But if any thing remains, (as in the 2d example) it muſt be plac'd as a new *numerator* over the given *denominator*.

EXAMPLES.

Reduce $\frac{32}{8}$ to its equivalent whole number.

$$\begin{array}{l} 8)32(4 \text{ facit.} \\ \quad\ \underline{} \\ \quad\ \ 0 \end{array}$$

Reduce $\frac{45}{6}$ $\quad 6)45(7\frac{3}{6}$ *facit.*

$$\begin{array}{l} \quad\ 42 \\ \quad\ \overline{\ \ 3\ } \end{array}$$

This being only the reverſe of the foregoing rule, the ſame reaſon ſtill holds.

Fourth, To reduce a compound fraction to a ſingle one, *i. e.* a fraction of a fraction, to a fraction of an unit.

R U L E.

Multiply all the *numerators* together for a *numerator*, and all the *denominators* for a *denominator*.

EXAMPLES.

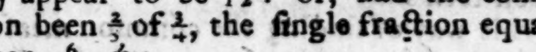
Reduce $\frac{2}{3}$ of $\frac{1}{4}$ to a ſingle fraction, *facit* $\frac{2}{12}$.

Reduce $\frac{4}{5}$ of $\frac{6}{8}$ of $\frac{7}{9}$ to a ſingle fraction.

$$\begin{array}{r}4\\6\\\hline 24\\7\\\hline 168\end{array}\qquad\begin{array}{r}5\\8\\\hline 40\\9\\\hline 360\end{array}\qquad \frac{168}{360}\ \textit{facit.}$$

So alſo $\frac{3}{8}$ of $\frac{9}{10}$ is $\frac{27}{80}$.

The reaſon of this operation will beſt appear by repreſenting the unit or whole number by a line, which, according to the firſt example, muſt be ſuppoſed to be divided into 4 parts, and each of thoſe parts again into 3 ſmaller parts; thus,



Then, as $\frac{1}{4}$ of the unit will denote 1 of the larger diviſions, ſo $\frac{2}{3}$ of that fourth muſt ſignify only 2 of the leſſer diviſions; conſequently if I would expreſs what part of the whole line $\frac{2}{3}$ of $\frac{1}{4}$ is, it will plainly appear to be $\frac{2}{12}$: or, had the compound fraction been $\frac{2}{3}$ of $\frac{3}{4}$, the ſingle fraction equal to it had been $\frac{6}{12}$, *&c.*

Fifth, To reduce a fraction into its loweſt terms.

R U L E.

Divide the *numerator* and *denominator* by any figure, ſo that nothing may remain, and the quotients will be a new fraction of the ſame value with that given.

The rule generally given for finding the greateſt common meaſure, or number, to divide your given fraction by, is this,

Divide the *denominator* by the *numerator*, and the *numerator* by the *remainder*, if any. So continuing to make the laſt diviſor the dividend, and the remainder the diviſor, till nothing remains, the laſt diviſor will be the greateſt common meaſure, or number, by which you can divide your fraction.

EXAMPLE.

What is the greateſt common meaſure by which $\frac{208}{684}$ can be divided?

```
208)684(3
    624
    ---
     60)208(3
        180
        ---
         28)60(2
            56
            --
 Anſwer,  4)28(7
            28
            --
             0
```

$\frac{208}{684}$ then being divided by 4, gives $\frac{52}{171}$, which are its loweſt terms.

But this way of finding the common meaſure, is too tedious, often making more work than it ſaves. Obſerve therefore theſe more practical directions.

If you cannot at once diſcover the greateſt number you may divide by, if both your *numerator* and *denominator* are even numbers, you may always halve them, and often that way reduce your fraction to advantage.

Thus the fraction given in the foregoing example, divided twice by 2, gives, as before, $\frac{57}{171}$.

$$2\ \frac{228}{684}\Big|\frac{114}{342}\Big|\frac{57}{171},$$

and $\frac{192}{576}$ may be reduc'd, by continual halving, to $\frac{3}{9}$,

$$2\ \frac{192}{576}\Big|\frac{96}{288}\Big|\frac{48}{144}\Big|\frac{24}{72}\Big|\frac{12}{36}\Big|\frac{6}{18}\Big|\frac{3}{9}.$$

Alſo, when both your *numerator* and *denominator* have cyphers on the right hand, you may abbreviate the fraction, by ſtriking off an equal number from both;

Thus, $\frac{45}{96}\Big|\frac{00}{00}$ or, $\frac{825}{960}\Big|\frac{00}{00}$:

Or, if the right hand figures of your *numerator* and *denominator*, are both fives, or one a five and the other a cypher (as the remainder of the laſt example) you may always divide them by 5;

thus, $\frac{825}{960}$, divided by 5,
is $\frac{165}{192}$ equal to $\frac{825}{960}$.

So alſo is $\frac{125}{365}$ reduc'd to $\frac{25}{73}$.

Thus, in the Rule of Three, we abbreviate our ſtatings by the ſame rule, and on the ſame reaſon; for the ſecond and third numbers multiply'd together, are as a *numerator* to the firſt number, their *denominator*.

For the reaſon of this rule, ſee the 2d note.

Sixth, To reduce fractions of divers denominations, into one of the ſame value.

R U L E.

Multiply each of the given *numerators* into all the given *denominators*, except its own, and the ſeveral products will be ſo many new *numerators*, whoſe common *denominator* muſt be the product of all the given *denominators* multiply'd together.

EXAMPLES.

Reduce $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{6}{10}$, into one denomination.

To do which,

Firſt, The *numerator* 3, muſt be multiply'd by the *denominators* 8 and 10, which will produce 240

Secondly, the *numerator* 5, multiply'd by the *denominators* 4 and 10, will produce 200

Thirdly, The *numerator* 6, multiply'd by the *denominators* 4 and 8, will produce 192

New Numerators.

Then the three *denominators* 4, 8, 10, multiply'd together, will give 320 for a common *denominator*.

See the work.

3	5	6	4
8	4	4	8
24	20	24	32
10	10	8	10
240	200	192	320
1ſt *Numer.*	2d *Numer.*	3d *Numer.*	*Denominator.*

$\frac{240}{320}$ $\frac{200}{320}$ $\frac{192}{320}$

So $\frac{5}{10}$, $\frac{3}{8}$, $\frac{4}{12}$, and $\frac{9}{15}$, reduc'd into one denomination, will be $\frac{7200}{14400}$, $\frac{5400}{14400}$, $\frac{4800}{14400}$, $\frac{8640}{14400}$.

See the work.

5	3	4	9
8	10	10	10
40	30	40	90
12	12	8	8
480	360	320	720
15	15	15	12
2400	1800	1600	8640
480	360	320	
7200	5400	4800	
1ſt *Numer.*	2d *Numer.*	3d *Numer.*	4th *Numer.*

10
8
80
12
960
15
4800
960
14400
Denominator.

The foregoing is the common rule for reducing fractions of ſeveral denominations into one: it may alſo be done by firſt finding the common *denominator* as before directed, and then to produce the new *numerators* dividing the ſaid common *denominator* ſeverally by each of the given *denominators*, and multiplying the quotients by the reſpective *numerators*; the products will be the ſeveral new *numerators*.

But, as by abbreviation, fractions are often reduc'd into much lower terms than they are at firſt given in; ſo here, a common *denominator* may ſometimes be found much ſmaller than that ariſing by the preceeding method. The rule therefore for diſcovering the leaſt common *denominator* to fractions of different denominations, is as follows, *viz.*

RULE.

Having firſt by the fifth ſort of reduction, reduc'd all your given fractions to their loweſt terms, cancel or ſtrike out all ſuch of the *denominators* as are *aliquot* parts of others, then drawing a line under all the *denominators*, if two or more can be divided by any ſuch number, ſo that nothing may remain, divide by ſuch number; and place the quotients under the line, and with them, in their proper places, all ſuch of the *denominators* as would not admit of ſuch diviſion. If any of the ſaid numbers or *denominators* ſo brought down, are capable of further abbreviation, draw another line, and proceed as before, ſo continuing as low as poſſible; then multiply continually, one into another, the ſeveral diviſors, and the quotients ariſing, or numbers remaining under the laſt line, and their product will be the leaſt common *denominator*. The new *numerators* to which may be found, as before, by dividing the ſaid common *denominator*, by each given *denominator*, and multiplying the quotients by their reſpective *numerators*.

EXAMPLE.

Reduce $\frac{3}{\cancel{4}}$ $\frac{5}{8}$ $\frac{6}{10}$ into one denomination.

$$\begin{array}{l} 2 \\ \hline 4\ \ 5 \end{array}$$

Anſwer, $\frac{30}{40}$ $\frac{25}{40}$ $\frac{24}{40}$

The common *denominator* 40 is thus produc'd.

Firſt, The *denominator* 4, being an *aliquot* part of 8, is cancell'd, then the *denominators* 8 and 10, are divided by 2. *Laſtly*, The diviſor $2 \times 4 \times 5 = 40$.

The reaſon of this ſixth ſort of reduction, is evident from the 2d of the preceding notes; for as both the *numerator* and *denominator* of each given fraction are equally multiply'd by all the other *denominators*, conſequently the new fractions thence ariſing, muſt be equal to the fractions given.

Seventh, To value a fraction, or reduce it to the known parts of an integer.

RULE.

Multiply the *numerator* by the next inferior denomination, and divide the product by the *denominator*, the quotient ſhews the parts ſought, and the *remainder* becomes a new *numerator* to the given *denominator*; which muſt ſtill be valued by the ſame rule, proceeding till you have brought it into the leaſt known parts of the integer.

EXAMPLES.

1ſt. What's the value of $\frac{3}{5}$ of a pound ſterling?

```
   3
  20
 ----
5) 60(
    12 Shillings.
```

2*d.* What's the value of $\frac{13}{49}$ of a pound Sterling?

```
       13
       20
      ———— s.  d.  qua.
   49)260(5   3   2 34/49 facit.
      245
      ————
       15
       12
      ————
   49)180(3
      147
      ————
       33
        4
      ————
   49)132(2
       98
      ————
       34
```

3*d.* What hundreds, quarters, and pounds, are contain'd in $\frac{53}{94}$ of a ton?

```
       53
       20
      ———— C.  qr.  lb.
  94)1060(11  1   2 92/94 facit.
     94
    ————
     120
      94
    ————
      26
       4
    ————
  94)104(1
      94
    ————
      10
      28
    ————
  94)280(2
     188
    ————
      92
```

There will be no difficulty in accounting for this rule, if we conſider but the particular working of any one example: Thus in the 1ſt, $\frac{3}{5}$ of a pound Sterling are given to be valued: Now as 20*s.* make a pound, ſo conſequently any part of a pound muſt be 20 times as great a part of a ſhilling; therefore $\frac{3}{5}$ of a pound make $\frac{60}{5}$ of a ſhilling; which being an improper fraction, its *numerator* is divided by its *denominator*, to find the units or whole numbers (which in this caſe muſt be ſhillings) contain'd in it; according to the directions of the 3d ſort of *Reduction*.

By this rule are *remainders* of ſtatings in the *Rule of Three*, &c. valued. See *page* 26.

ADDITION *of* FRACTIONS.

WHEN the given fractions are of one denomination, all you have to do, is, to add their *numerators* together, and to place the ſum for a new *numerator*, over the *denominator* given; thus, $\frac{1}{6}$, $\frac{4}{6}$, and $\frac{3}{6}$, added together, make $\frac{8}{6}$, or $1\frac{2}{6}$.

But when amongſt the given ſums, there are either compound fractions, or ſingle ones of different denominations, they muſt be prepar'd by reduction, before they can be added.

EXAMPLES.

1*ſt*. What's the ſum of $\frac{3}{8}$, $\frac{2}{9}$, and $\frac{4}{7}$?

According to the 6th ſort of reduction, the given fractions being brought into one denomination, they will be $\frac{189}{504}$, $\frac{112}{504}$, $\frac{288}{504}$; the *numerators* of which being added, the ſum is $\frac{589}{504}$, or $1\frac{85}{504}$.

2*d*. Add $\frac{3}{4}$ of $\frac{1}{8}$, and $\frac{9}{10}$ of $\frac{6}{7}$ and $\frac{5}{8}$ together.

Reduce the compound fractions to ſingle ones, by the 4th rule of reduction; and inſtead of $\frac{3}{4}$ of $\frac{1}{8}$, and $\frac{9}{10}$ of $\frac{6}{7}$, you will have $\frac{9}{32}$ and $\frac{54}{70}$, to add to your $\frac{5}{8}$; all which ſingle fractions being reduc'd, as before, to one denomination, they will be $\frac{5040}{17920}$, $\frac{13824}{17920}$, and $\frac{11200}{17920}$; which added, makes $\frac{30064}{17920}$, or 1 $\frac{12144}{17920}$.

3*d*. Add 4 $\frac{1}{2}$ and 3 $\frac{1}{6}$ and $\frac{2}{7}$ together.

When mix'd numbers are given to be added, reduce only their fractional parts to one denomination, and add their ſum to the total of the whole numbers.

$$\begin{array}{lllr} \frac{42}{84} & \frac{14}{84} & \frac{24}{84} & 4 \text{——} 42 \\ & & & 3 \text{——} 14 \\ & & & 24 \\ \hline & & & 80 \\ & & \textit{facit}\ 7 & \overline{84} \end{array}$$

More EXAMPLES.

Add 8 *l*. $\frac{1}{2}$ and 3 *l*. $\frac{9}{10}$ and $\frac{17}{25}$ together.
Facit 13 *l*. $\frac{4}{50}$.

[What's the ſum of $\frac{3}{4}$ of $\frac{7}{8}$, and 4 *l*. $\frac{1}{2}$ and $\frac{18}{30}$?
Anſwer, 5 *l*. $\frac{372}{480}$.

SUBSTRACTION *of* FRACTIONS.

SUBſtraction of Fractions is alſo nothing (after the given fractions are prepar'd by reduction) but taking one *numerator* from the other.

EXAMPLES.

EXAMPLES.

1ſt. *From* $\frac{3}{4}$ *take* $\frac{4}{9}$ 27

16

—

prepar'd $\frac{27}{36}$ $\frac{16}{36}$ $\frac{11}{36}$ *Remainder.*

2d. *From* 5 $\frac{2}{3}$ *take* $\frac{9}{10}$.

Firſt, The mix'd number 5 $\frac{2}{3}$ being reduc'd to the improper fraction $\frac{17}{3}$, by the ſecond rule of reduction, proceed as before.

From $\frac{17}{3}$ *take* $\frac{9}{10}$, *or prepared.*
From $\frac{170}{30}$ *take* $\frac{27}{30}$ *Rem.* $\frac{143}{30}$ *or* 4 $\frac{23}{30}$.

Or without reducing the mix'd number to an improper fraction, make only the fractional parts of one denomination, and you will then have $\frac{27}{30}$ to take from $\frac{20}{30}$; which, becauſe you cannot do, you muſt borrow an unit, *i. e.* $\frac{30}{30}$, then $\frac{27}{30}$ from $\frac{30}{30}$, there will remain $\frac{3}{30}$; to which adding the $\frac{20}{30}$, the remainder will then be $\frac{23}{30}$; and the unit borrowed being deducted from the whole number 5, there will remain in all 4 $\frac{23}{30}$, as before.

More EXAMPLES.

From 5 *l.* $\frac{7}{8}$ *take* $\frac{9}{10}$ of $\frac{5}{6}$ *Rem.* 5 *l.* $\frac{1}{8}$.
From $\frac{19}{20}$ *take* $\frac{1}{7}$ of $\frac{2}{3}$. *Rem.* $\frac{359}{420}$.

MULTIPLICATION *of* FRACTIONS.

IF the numbers given, are whole, or mix'd, they muſt firſt be brought into improper fractions; or if they are compound fractions, they muſt be reduc'd to ſingle ones; but if the numbers given, are both either proper or improper ſingle fractions you have only to multiply the two given *numerators* together for a new *numerator*, and the given *denominators* for a *denominator*.

EXAMPLES.

1*ſt*. Multiply $\frac{4}{8}$ by $\frac{3}{6}$ *facit* $\frac{12}{48}$, or by abbreviation $\frac{1}{4}$.

2*d*. Multiply $\frac{9}{6}$ by $\frac{7}{4}$ *facit* $\frac{36}{42}$ or $\frac{6}{7}$.

3*d*. Multiply 4 by $\frac{3}{8}$. 4 by the firſt ſort of reduction, reduc'd to a fraction, is $\frac{4}{1}$, which multiply'd by $\frac{3}{8}$, makes $\frac{12}{8}$, or 1 $\frac{1}{2}$.

4*th*. Multiply 7 $\frac{1}{2}$ by 8 $\frac{2}{3}$. Theſe mix'd fractions, reduc'd by the 2d rule of reduction to improper ones, make $\frac{15}{2}$ and $\frac{26}{3}$, which multiply'd together, make $\frac{390}{6}$, or 65.

Two things require explanation in multiplication of fractions, 1*ſt*. Why, when the multiplier is a proper fraction, the product is always leſs than the multiplicand?

2*d*. Why the *denominators* are multiply'd as well as the *numerators*, whereas in addition, we only find the ſum of the *numerators*?

Now the reaſon why a number multiply'd by a fraction is decreas'd, will appear by conſidering that as an unit is no multiplier, *i. e.* that any number multiply'd by 1, remains ſtill the ſame, as once 4 is 4, *&c.* ſo to multiply any thing by a fraction, which is but a part of 1, muſt conſequently produce bnt ſuch a proportionable part of the multiplicand, as ſuch fraction is of an unit. Thus 4 multiply'd by $\frac{1}{2}$, produces but 2, the half of 4; which 4 being made a fraction by the 1ſt rule of reduction, is $\frac{4}{1}$; ſo that the numbers reſolve themſelves into a compound fraction; and the product of 4, multiply'd by $\frac{1}{2}$, is plainly the $\frac{1}{2}$ of $\frac{4}{1}$; which by the rule for reducing a compound fraction, makes $\frac{4}{2}$, or 2; the reaſon therefore of the work is clear, from the explanation of the 4th ſort of reduction.

More EXAMPLES.

Multiply 9 $\frac{1}{2}$ by $\frac{2}{6}$, *facit* 3 $\frac{1}{6}$.
Multiply 3 *d.* $\frac{3}{4}$ by 3 *d.* $\frac{3}{4}$, *facit* 14 *d.* $\frac{1}{16}$.

DIVISION *of* FRACTIONS.

WHEN you have made the ſame preparation of your numbers as directed in multiplication, multiply the *denominator* of your diviſor by the *numerator* of your dividend, for a *numerator*; and the *denominator* of your dividend, by the *numeraaor* of your diviſor, for a *denominator*.

EXAMPLES.

1*ſt*. Divide $\frac{7}{9}$ by $\frac{3}{8}$, *facit* $\frac{56}{27}$, or 2 $\frac{2}{27}$.

2*d*. Divide 7 by $\frac{3}{4}$, prepar'd by the firſt rule of reduction, it will be $\frac{7}{1}$ by $\frac{3}{4}$, *facit* $\frac{28}{3}$, or 9 $\frac{1}{3}$.

3*d*. Divide 4 $\frac{1}{2}$ by 2 $\frac{3}{8}$, reduc'd by the 2d rule, it is $\frac{9}{2}$ by $\frac{19}{8}$, which, divided, gives $\frac{72}{38}$, or 1 $\frac{34}{38}$.

4*th*. Divide $\frac{2}{3}$ of $\frac{1}{9}$ by $\frac{4}{8}$ of $\frac{7}{8}$, reduc'd by the 4th rule, $\frac{2}{27}$ by $\frac{28}{64}$, *facit* $\frac{128}{756}$.

There is nothing in the whole practice of fractions, that more requires an explanation, than the manner of performing diviſion, ſo different from that of whole numbers. Now, as the effect of diviſion is the finding how often one number is contain'd in another; or, which is the ſame thing, what part of the dividend the diviſor is; ſo it is plain, that if the diviſor is unity or 1, the quotient muſt be equal to the dividend, conſequently, in what proportion ſoever the diviſor exceeds unity, in ſuch proportion muſt the quotient be leſs than the dividend, and in what proportion ſoever the diviſor is leſs than unity, by the ſame proportion muſt the quotient exceed the dividend. Thus, ſuppoſe the number 4 was given for a dividend, if 1 is propos'd for its diviſor, the quotient will be alſo 4; but if we make the diviſor 2, twice the former diviſor, the quotient will be but 2, half the former quotient. Again, let the diviſor be $\frac{1}{2}$, *i. e.* half of 1, and the quotient will then be found to be $\frac{8}{1}$, or 8, twice as much as the firſt quotient, as the diviſor is here but half the firſt diviſor. From whence appears the reaſon,

1ſt. Why dividing a number by a proper fraction gives a quotient greater than the dividend: as multiplying a number by a fraction was before prov'd to give a product leſs than the multiplicand.

2d. Why the operation is to be perform'd in the manner directed; for, as the greater the *denominator* is, the leſs is the value of the fraction, and conſequently that the quotient muſt encreaſe in the ſame proportion, is a clear reaſon for multiplying the *numerator* of the dividend by the *denominator* of the diviſor, for the *numerator* of the quotient; ſo in like manner, as the increaſe of the *numerator* of a fraction is the increaſe of the value of ſuch fraction, it is therefore as plain, the value of the quotient muſt be diminiſhed in the ſame proportion, by the increaſe of its *denominator*, which is certainly effected by multiplying the *denominator* of the dividend, by the *numerator* of the diviſor. Thus $\frac{5}{6}$ divided by $\frac{1}{4}$, gives $\frac{20}{6}$; but divided by $\frac{2}{4}$, gives but $\frac{20}{12}$, the value of the quotient ſtill decreaſing in the ſame proportion with the increaſe of the value of the diviſor.

More EXAMPLES.

Divide $\frac{7}{8}$ by $\frac{1}{3}$. *Quot.* 2 $\frac{5}{8}$.
Divide 9 $\frac{1}{2}$ by $\frac{2}{5}$ of $\frac{3}{4}$. *Quot.* 31 $\frac{3}{4}$.

The DIRECT RULE *of* THREE IN VULGAR FRACTIONS.

AFter the numbers are prepared by reduction, ſtate and work the queſtion by the rules given in whole numbers.

EXAMPLES.

1*ſt*. What will $\frac{1}{2}$ a pound of ſnuff coſt, if 22 ℔ $\frac{1}{4}$ of the ſame come to 7 *l.* $\frac{1}{3}$?

℔. *l.* ℔.
If 22 $\frac{1}{4}$ *coſt* 7 $\frac{1}{3}$, *what will* $\frac{1}{2}$?

Prepar'd by the 2d rule of reduction, it will ſtand thus,

If $\frac{89}{4}$ . . $\frac{22}{3}$ . . $\frac{1}{2}$?

then the ſecond and third numbers multiply'd together, produce $\frac{22}{6}$; which divided by $\frac{89}{4}$, gives $\frac{88}{534}$; which, valu'd by the 7th rule of reduction, is 3 *s.* 3 *d.* 2 *qu.* $\frac{108}{534}$.

2*d*. Bought 87 gallons and an $\frac{1}{2}$ of brandy for 40 *l.* at the ſame rate, what would $\frac{1}{2}$ a gallon coſt?

Gall. *l.* *Gall.*

If 87 $\frac{1}{2}$ *coſt* 40, what will $\frac{1}{2}$?

prepar'd $\frac{175}{2}$ —— $\frac{40}{1}$ —— $\frac{1}{2}$

l. *s. d. qu.*

then $\frac{40}{1} \times \frac{1}{2} = \frac{40}{2} \cdot \frac{40}{2} \div \frac{175}{2} = \frac{80}{350} = \frac{8}{35}$, or 4 6 3 $\frac{1}{7}$

Another RULE.

When your queſtion is ſtated, and your numbers prepar'd, multiply the *denominator* of the firſt number by the *numerators* of the ſecond and third, and place the product for a *numerator:* Then multiply the *numerator* of the firſt number by the *denominators* of the ſecond and third, and place the product for its *denominator*; and the new fraction ſo found, is the anſwer of the queſtion. Take the ſtating of the laſt queſtion for an

EXAMPLE.

If $\frac{175}{2} \cdot\cdot \frac{40}{1} \cdot\cdot \frac{1}{2}$ *facit* $\frac{80}{350}$.

For 2, the *denominator* of the firſt number, multiply'd by 40 and 1, the *numerators* of the ſecond and third numbers, is 80.

And 175, the *numerator* of the firſt, multiply'd by 1 and 2, the *denominators* of the ſecond and third is 350. Theſe products plac'd fractionally, make $\frac{80}{350}$, as before.

3*d*. What will 82 C. of Sugar come to, if $\frac{3}{4}$ of $\frac{3}{8}$ of a C. coſts $\frac{9}{10}$ of a pound Sterling?

C. *l.* *C.*

If $\frac{3}{4}$ *of* $\frac{3}{8}$ *coſt* $\frac{9}{10}$, *what will* 82?

$\frac{9}{32} \cdot\cdot \frac{9}{10} \cdot\cdot \frac{82}{1}$

Facit $\frac{23616}{90}$ *or* 262 *l.* 8 *s.*

More

More EXAMPLES.

What will $\frac{1}{4}$ of an ounce of ſnuff coſt, if the lb comes to 8*s*. $\frac{1}{3}$?

Anſwer, 1 *d*. 2 *qua*. $\frac{1}{4}$.

At 1 *d*. $\frac{1}{2}$ *per* ounce, what will 5 C. $\frac{1}{2}$ come to?

Anſwer, 61 *l*. 12 *s*.

The INDIRECT RULE *of* THREE
IN
VULGAR FRACTIONS.

HEre alſo you have only to obſerve the directions given in whole numbers:

Or this RULE.

After the numbers are ſtated and prepared, multiply the *numerators* of the firſt and ſecond numbers, by the *denominator* of the third, for a *numerator*; and the *denominators* of the firſt and ſecond, by the *numerator* of the third number, for a *denominator*; and the fraction ſo found, is the *Anſwer*. See both ways in the example.

EXAMPLE.

How many yards of ſtuff, $\frac{1}{2}$ yard wide, are equal to 36 $\frac{1}{4}$ yards of $\frac{3}{4}$ wide?

$$\textit{If } \frac{3}{4} \text{———} 36\tfrac{1}{4} \text{———} \frac{1}{2}$$

$$\textit{Prepar'd } \frac{3}{4} \cdots\cdots \frac{145}{4} \cdots\cdots \frac{1}{2}$$

$$\textit{Then } \frac{3}{4} \times \frac{145}{4} = \cdot \frac{435}{16} \cdot \frac{435}{16} \div \frac{1}{2} = \frac{870}{16} \textit{ or } 54\tfrac{3}{8}.$$

So alſo 3, 145, and 2, the *numerators* of the firſt and ſecond numbers, and the *denominator* of the third, multiply'd together, is 870.

And 4, 4, and 1, the *denominators* of the firſt and ſecond, and *numerator* of the third, multiply'd together, produce 16.

Theſe numbers fractionally plac'd, make $\frac{870}{16}$, as before.

More EXAMPLES.

If *A*. lends *B*. 25 *l*. $\frac{1}{3}$ for 6 months $\frac{1}{2}$, how long may he keep 10 *l*. $\frac{1}{4}$ of *B's*, to requite himſelf?

Anſwer, 16 months $\frac{16}{240}$.

If *A*. keeps 100 *l*. $\frac{1}{6}$ of *B*'s 4 months $\frac{1}{3}$, what ſum muſt *A*. lend *B*. for 2 years $\frac{1}{2}$ to requite him?

Anſwer, 14 *l*. $\frac{253}{540}$.

The End of the Firſt Part.

THE SECOND PART:

TREATING OF

Decimal Arithmetick.

THIS part of Arithmetick is very compendious, and therefore very uſeful, eſpecially in calculations of intereſt, valuing annuities, *&c*. But by reaſon ſome fractions of coin, weight, and meaſure, cannot be exactly expreſs'd by decimal parts, but that the numbers will often circulate, it is not always convenient to work with them. Mr. *Cunn* hath, indeed, in his late treatiſe on fractions, very curiouſly ſhewn how they may be us'd with leaſt loſs; but ſince ſuch exactneſs deſtroys their brevity, I cannot but think, in ſuch caſes, vulgar fractions preferable.

CHAP. I.

Of NUMERATION, ADDITION, SUBSTRACTION, MULTIPLICATION, *and* DIVISION *of* DECIMALS.

NUMERATION.

IN DECIMAL FRACTIONS, the *integer*, or whole thing, (whether it be time, coin, weight, or meaſure, as one year, one pound Sterling, one pound weight, *&c.*) is ſuppos'd to be divided into tenths, or ten equal parts; and thoſe parts again into tenths, and ſo on *ad infinitum*.

So that the *denominator* of a decimal fraction being always known to conſiſt of an unit, with as many cyphers as the *numerator* hath places, is ſeldom ſet down; the parts being only diſtinguiſh'd from whole numbers, by a point or comma prefix'd; thus, ,5 ſtands for $\frac{5}{10}$, ,25 for $\frac{25}{100}$, ,123 for $\frac{123}{1000}$, *&c.*

But the different value of the ſeveral places, will more plainly appear from the following table.

TABLE.

Whole Numb.							Decimal Parts.						
	6	5	4	3	2	1,	2	3	4	5	6	7	
&c.	C Thouſands.	X Thouſands.	Thouſands.	Hundreds.	Tens.	Units.	Tenths, or Parts of 10	Parts of 100	Parts of 1000	Parts of 10000	Parts of 100000	Parts of 1000000	*&c.*

from whence it is evident,

1*ſt*. That as whole numbers increaſe by a tenfold proportion towards the left hand, from the unit's place; ſo decimal parts decreaſe, in the ſame proportion, towards the right.

2*d*. That therefore cyphers, plac'd before decimal parts, decreaſe their value, by removing them farther from the comma, or unit's place.

$$\text{thus,}\left\{\begin{array}{l} ,5, \text{ is } 5 \text{ parts of ten, or } \frac{5}{10}. \\ ,05, \text{ is } 5 \text{ parts of an hundred, or } \frac{5}{100}. \\ ,005, \text{ is } 5 \text{ parts of a thouſand, or } \frac{5}{1000}. \end{array}\right.$$

3*d*. But that cyphers, after decimal parts, alter not their value, becauſe they do not change their places.

For ,5, ,50, ,500, *&c*. are each but $\frac{5}{10}$ of an unit, or 1; the 5 ſtill keeping its firſt place.

Theſe things underſtood, the reſt is eaſy: But let the reader be ſure he does underſtand them before he proceeds.

ADDITION.

HAving ſet down your propoſed numbers, units under units, tens under tens, and parts under parts of like value, add them as if they were all whole numbers, ſeparating ſo many decimal parts in the total, as were in any of the given numbers.

EXAMPLES.

Add 5,42, and 38,583, and 421,06, and ,8345 together.

Theſe numbers, 5,42
rightly plac'd, 38,583
will ſtand thus: 421,06
,8345

Total, 465,8975

What's the ſum of 4,06, and 21,734, and 523,8642
l. l. l.
21,734
4,06

Anſwer, 549,6582 *l.*

SUBSTRACTION.

Having plac'd the numbers as directed in addition, work ſtill as if they were all whole numbers.

EXAMPLES.

l. l.
From 839,38275 take 10,49561
10,49561

828,88714 *Remainder.*

Yards. *Yards.*
Take 23,279 *from* 451,15842
23,279

Remains 427,87942

MULTI-

MULTIPLICATION.

HEere the numbers are to be ſet down, (without regard to the value of their places) and work'd in all reſpects the ſame as in whole numbers; only obſerve to ſeparate ſo many places of decimal parts on the right hand of the product, as were in both multiplicand and multiplier.

EXAMPLES.

Multiply 2,735641
by ,54382

	5471282
21	885128
82	06923
1094	2564
13678	205
1,4876	9628862

Multiply 73,82564365
by 8,7434592

	14765128730
	66443079285
3	6912821825
29	530257460
221	47693095
2953	0257460
51677	950555
590605	14920
Product 645,491	503167514080

A compendious way to multiply numbers that have many decimal places; by omitting all the figures to the right of the perpendicular lines in the examples above.

RULE.

Place one of the numbers juſt as it is given, for a multiplicand, and under that decimal place which you would have be the laſt in the product, ſet the unit's place of your other number; then reverſing all the other figures, multiply with them in their order, beginning always with that figure of the multiplicand, which ſtands over the number you work with; having alſo reſpect to the increaſe you think would have been brought from the omitted figures on the right hand, had they been multiply'd.

Note, the uſual allowance for ſuch omitted figures is as follows, *viz.* If the next figure on the right hand of that you begin with in the multiplicand, multiply'd into the figure of the multiplier you are working with, gives a product betwixt 5 and 15, carry 1; if the product be above 15, and leſs than 25, carry 2; and if it ariſe to any number betwixt 25 and 35, carry 3, *&c.*

To prove the certainty of the rule, we will take the ſame examples before given.

EXAMPLES.

Multiply 2,735641 by ,54382, and produce only four places of decimals.

2,735641
28345,0

13678 *the Increaſe here carried is* 3 *for* 5 × 6
1094 *the Increaſe here is* 2 *for* 4 × 5
82 *the Increaſe here is* 1 *for* 3 × 3
22 *the Increaſe here is* 6 *for* 8 × 7

1,4876

Note, There being no units in the multiplier, its place is ſupply'd with a cypher.

Multiply 73,82564365 by 8,7434592, and produce but 3 places of decimals.

73,82564365
2954347,8

590605
51678
2953
222
29
4 *the Increaſe carried for* 5 × 7

645,491

Note, To multiply any decimal number by 10, 100, 1000, *&c.* is only to remove the mark of ſeparation ſo many places towards the right hand as there are cyphers.

$$\text{Thus } 8,3564537 \text{ multiply'd by} \left\{\begin{array}{l}10,\\100,\\1000,\\10000,\end{array}\right\} \text{is} \left\{\begin{array}{l}83,564537\\835,64537\\8356,4537\\83564,537, \&c.\end{array}\right.$$

DIVISION.

THIS rule alſo being work'd the ſame as in whole numbers, the only difficulty is to value the quotient; which may be done by either of the following general rules, *viz.*

RULE 1.

The firſt figure in the quotient, is always of the ſame value with that figure of the dividend, which anſwers, or ſtands over the place of units in the diviſor. Or,

RULE 2.

The quotient muſt always have ſo many decimal places, as the dividend has *more* than the diviſor.

Note, 1*ſt*. If the diviſor and dividend have both the ſame number of decimal parts, (as in the ſecond example) the quotient will be all a whole number.

2*d*. If the dividend hath not ſo many places of decimals as are in the diviſor, (as in example the 3d) ſo many cyphers muſt be annex'd to the dividend, as will make them equal; and the quotient will then, as before, be all a whole number.

3d. But if when diviſion is done, the quotient has not ſo many figures as it ſhould have places of decimal parts, (as in the laſt example) ſo many cyphers muſt be prefix'd, as there are places wanting.

EXAMPLES.

1ſt. Divide 10,55439 by 4,569

4,569)10,55439(2,31
14163
4569
...

Here, according to the firſt rule, 4, the unit's place of the diviſor, ſtanding under units, or the firſt place of whole numbers in the dividend; 2, the firſt figure plac'd in the quotient, muſt alſo be made the firſt place of whole numbers, by placing the point of ſeparation immediately after it.

Or, according to the ſecond rule, there being five places of decimals in the dividend, and but three the diviſor; two places muſt be ſeparated off in the quotient, to make up the number.

Ex. 2d. Divide 85643,825 by 6,321

6,321)85643,825(13549
22433
34708
31032
57485
·596

Here 6, the place of units in the diviſor, ſtanding under 8, the fifth place of whole numbers in the
the

the dividend 1, the firſt figure of the quotient, muſt alſo ſtand in the ſame place, which makes all the quotient whole numbers.

And ſo it muſt alſo be by the ſecond rule, becauſe the dividend has no more places of decimals than the diviſor.

Ex. 3. Divide 7382,54 by 6,4352.

6,4352)7282,5400(1147

Here there not being ſo many places of decimals in the dividend, as in the diviſor, two cyphers are added to make them even; and being ſo, the quotient is all a whole number, as before.

Ex. 4. Divide ,08516438 by 423

423),08516438(,00020133

··564

1413

·1448

179

In this example, the quotient conſiſting but of five figures, whereas there ſhould, by the rules, be eight places of decimal parts, three cyphers are prefix'd to make up the number, as was directed in *Note* the third.

There is alſo a way of contracting the work of a large diviſion, ſomewhat like that compendious method before taught of multiplying numbers of many places of decimals, and bringing out only ſo many in the product as are neceſſary for the anſwer required.

RULE.

Having conſider'd by the firſt rule given for valuing the quotient, the value of the firſt figure therein to be placed, and accordingly determin'd how many figures it is neceſſary it ſhould conſiſt of, you may from thence judge whether any, or how many of the right hand figures of your diviſor may be neglected, ſo that reſerving only ſo many figures of the dividend as are neceſſary for once anſwering the diviſor, cut off the reſt to the right hand as uſeleſs, and having ſet the proper figure in the firſt place of the quotient, work with it as uſual; then omitting the right hand figure of your diviſor, ſeek how often the other figures of your diviſor are contain'd in the remainder, which figure being enter'd in the quotient, and work'd with as uſual in diviſion, (with regard, however, to the carriage that would have been brought from the omitted figures, as before directed in multiplication) thus continuing to neglect the right hand figure of your diviſor every time you ſeek a new quotient figure, you will ſtill be able to divide the remainder left by the laſt ſubſtraction by the diviſor ſo leſſen'd, till you have brought out the determin'd number of figures in the quotient.

The following example, which is ſet down both at length, and the contracted way, will make all clear.

EXAMPLE.

```
3,4637528)95,4327|56463275(27,55183
            692750|56
          --------------
            261577|004
            242462|696
          --------------
           ·19114|3086
            17318|7640
          --------------
            ·1795|54463
             1731|87640
          --------------
              ·63|668232
               34|637528
          --------------
               29|0307047
               27|7100224
          --------------
               ·1|32068235
                1|03912584
          --------------
                ·|28155651
```

```
3,46375|28)95,4327|56463275(27,5518
 · · · · ·
 -    -        692750
              --------
               261577
               242463
              --------
              ·19114
               17319
              --------
               ·1795
                1732
              --------
                 ·63
                  35
              --------
                  28
                  26
              --------
                   2
```

And thus are all the figures on the right hand of the perpendicular line in the example work'd at length, ſav'd by working after this contracted manner.

As multiplying by 10, 100, 1000, *&c.* is only removing the ſeparating point of the multiplicand, ſo many places as there are cyphers in the multiplicator; ſo, to divide by 10, 100, 1000, *&c.* is only removing the mark of ſeparation, after the ſame manner, towards the left hand.

Thus 83564,537 divided by	10,	is	8356,4537
	100,		835,64537
	1000,		83,564537
	10000,		8,3564537

Theſe examples being, I hope, ſufficient to make diviſion plain, we will now proceed to *Reduction of Decimals*.

CHAP. II.

Of REDUCTION *of* DECIMALS.

THERE are two ſorts of *Reduction of Decimal Fractions*. 1ſt. changing a vulgar fraction to a decimal. 2. valuing a decimal by the known parts of an integer.

1ſt. To reduce or change a vulgar fraction to a decimal.

RULE.

Place cyphers at pleaſure after the *numerator*, and divide by the *denominator*.

EXAMPLES.

Reduce $\frac{1}{4}$ to a decimal 4)1,00(,25 *facit.*

What decimal parts are equal to $\frac{3}{8}$?

8)3,000(,375 *facit.*

And ſo any part of coin, weight, meaſure, or time, *&c.* may be brought into a decimal, by firſt reducing it into a vulgar fraction, (which is done by placing the known parts of the integer as a denominator under the parts given) then annexing cyphers to the *numerator*, and dividing by the *denominator*, as before directed.

EXAMPLES.

EXAMPLES.

What decimal parts of a pound are equal to 16*s*?

i. e. $\frac{16}{20}$ 20)16,0(,8 *facit.*

What's the decimal of 9 *oz.* the inter a ℔. *Troy?*

i. e. $\frac{9}{12}$ 12)9,00(,75 *facit.*

Note, If the given parts are of ſeveral denominations, they may be reduc'd either by ſo many diſtinct operations as there are different parts, or by firſt reducing them into their loweſt denomination, and then one diviſion will ſerve. See both ways in the

EXAMPLES.

Reduce 14 *s.* 9 *d* $\frac{3}{4}$. into decimal parts of a pound Sterling: By the firſt way there will be theſe three vulgar fractions to be chang'd, $\frac{14}{20}$, $\frac{9}{240}$, and $\frac{3}{960}$.

20)14,00(,70

960)3,0000(,0031
1200
240

240)9,0000(,0375
1800
1200
00

,70
,0375
,0031
facit ,7406

s. *d.*
The other way. 14 9 $\frac{3}{4}$ *being reduc'd to farthings.*
12
177
4
will be 711
which divided by its denominator, 960)711,0000(,7406 *is produc'd as (before.*
3900
6000
240

What decimal parts of C. are equal to 3 *qrs.* 18 ℔?

By the firſt way:

$\frac{1}{4}$ and $\frac{18}{112}$ 4)3,00(,75
,1607

,9107 *facit.*

112)18,0000(,1607
·680
··800
16

By the ſecond way:

qrs. ℔.
3 18
28

102

---

112 *Denominator*

112)102,0000(,9107
·120
·800
16

There is alſo another way of reducing parts of different denominations, into decimals of the higheſt integer; thus,

R U L E.

Bring the loweſt parts into decimals of the next ſuperior denomination; and on the right hand of the decimal found, place the parts given of that ſaid next ſuperior denomination; ſo proceeding, till you bring out the decimal parts of the higheſt integer requir'd, by ſtill dividing the product by the next ſuperior *denominator*; for examples take thoſe before given, *viz.*

Reduce 14*s*. 9*d* $\frac{3}{4}$. into decimal parts of a pound Sterling.

$$\begin{array}{r|l} 4 & 3, \\ \hline 12 & 9{,}75 \\ \hline 20 & 14{,}8125 \\ \hline & {,}7406 \end{array}$$

Here the 3 farthings being divided by 4, gives 75, to which the 9*d*. being prefix'd, it becomes 9,75; which divided again by 12, quotes ,8125; before which the 14*s*. being alſo plac'd, it makes 14,8125; which being laſtly divided by 20, the quotient ,7406, is the anſwer.

What decimal parts of a C. are equal to 3 *qrs* 18 ℔?

$$28\begin{cases} 7 \\ 4 \end{cases}\begin{array}{|l} 18, \\ \hline 2{,}5714 \\ \hline \end{array}$$

$$\begin{array}{r|l} 4 & 3{,}6428 \\ \hline & {,}9107 \end{array}$$

Note, Inſtead of dividing by 28, which would be troubleſome, 'tis better to divide by 7 and 4, as in the example, 4 times 7 being 28.

Thus you ſee how eaſily the decimals, anſwering to any given parts of coin, weight, meaſure, &c. are found; and after the ſame manner are the following tables form'd.

But ſhillings, pence, and farthings, may more readily be reduc'd thus:

R U L E.

For the ſhillings, (if their number be even) ſet down their half in the firſt place of decimals, and let the ſecond and third places be fill'd up with the farthings contain'd in the remaining pence and farthings, always remembring to add one when the number is, or exceeds 25; but if the number of ſhillings be odd, the ſecond place of decimals muſt alſo be increas'd by 5.

Thus the decimal of 8 *s*. 5 *d* ½. will be ,422, 4 in the firſt place of decimals, ſtanding for 8 *s*. and 22 farthings being 5 *d* ½.

Again, 8 *s*. 6 *d* ½ will be ,427.

1 being added to 26, the number of farthings in 6 *d* ½, becauſe exceeding 25.

But, laſtly, 9 *s*. 6 *d* ½ will be thus expreſs'd by decimals, *viz*. ,477.

5 being added to the ſecond place, becauſe the number of ſhillings is odd, the decimal of 1 ſhilling being ,05.

The ſecond ſort of reduction, is to value a decimal by the known parts of its integer; which is thus done:

R U L E.

Multiply the decimal given, by the known parts of the next inferior denomination, and ordering the product as directed in multiplication, ſtill multiply the remaining decimals by the known parts of the next inferior denomination; thus proceeding 'till you have brought it into the leaſt known parts of the integer, the figures ſtanding on the left of the ſeperating points, will be the parts requir'd.

E X A M P L E S.

Ex. 1. What's the value of ,7691, the integer a pound Sterling, (*i. e.*) What ſhillings, pence, and farthings, are equal to ,7691 parts of a pound Sterling?

	,7691	*parts of a pound*
multip'y'd by	20	*the ſhillings in* 1 *l*.
produce ſhillings,	15,3820	*and parts of a ſhilling,*
which multiply'd by	12	*the pence in* 1 *ſhilling,*
make pence	4,5840	*and parts of a penny,*
which again multiply'd by	4	*the farthings in* 1 *d*.
give farthings	2,3364	*and par. of a farthing.*

So that the value of ,7691 decimal parts of a pound Sterling, is 15 *s.* 4 *d* ½.

Ex. 2. What ounces, pennyweights, and grains, are equal to ,84362 decimal parts of a pound *Troy*?

	,84362	*parts of a pound* Troy,
Multiply'd by	12	*the ounces in one pound,*
Give ounces	10,12344	*and parts of* 1 *ounce.*
Multiply'd by	20	*the pennyweights in* 1 *oz.*
Give pennyweights	2,46880	*and parts of* 1 *pennywt.*
Multiply'd by	24	*the grains in* 1 *penynwt.*
	187520	
	93760	
Give grains	11,25120	*and parts of* 1 *grain.*

The Anſwer *therefore is* 10 *oz.* 2 *dwts.* 11 *grs.*

But the decimal parts of a pound Sterling, may be thus valu'd at ſight:

RULE.

The figure ſtanding in the firſt place of decimals, doubled, gives ſhillings; but if the figure in the 2d place, is, or exceeds 5, one more muſt be added to their number; the ſecond figure (if under 5) or its exceſs, (if above 5) join'd with the third, are ſo many farthings; only remember to abate 1 if their number amounts to 25; or 2, if near 50.

The reaſon of this abatement is, that as 1000 is the *denominator* of every decimal conſiſting of 3 places, ſo conſequently by reckoning the figure ſtanding in the ſaid third place, as ſo many farthings, we thereby allow 1000 farthings to the pound, whereas indeed there are but 960, the overplus therefore being 40 in 1000, is certainly 4 in a 100, or 1 in 25, *&c.* according to the direction above.

EXAMPLES.

Ex. 1. What's the value of ,375 parts of a pound Sterling?

Anſwer, 7 *s.* 6 *d.*

The 3 doubled is 6 ſhillings, to which 1 *s.* being added for the 5, in the ſecond place, makes it 7 *s.* and 2 remaining, join'd with the 5 in the 3d place, being accounted 25 farthings, from which 1 being deducted, there remains 24 farthings, or 6 pence.

Ex. 2. What's the value of ,846 parts of a pound?

Anſwer, 16 *s.* 11 *d.*

The 8 doubled, making 16 *s.* and 2 abated from the 46 farthings, leaves 44 farthings, or 11 *d.*

Thus are ſhewn the ways both of finding the decimals anſwerable to any given parts of coin, weight, meaſure, *&c.* or the value of any given decimal in the known parts of its integer.

I have likewiſe annex'd TABLES ready calculated by the foregoing rules, anſwering to the ſame ends.

The uſe of which is very plain, one column in each table containing the number of ſhillings, ounces, drams, points, buſhels, days, hours, or any thing elſe you may want. And againſt the ſaid number, in the other column, the decimal parts anſwering. Thus againſt 3 pence in the firſt table, is found ,0125, and againſt 40 days in the fourth table, ,109589.

But if the number you want is not to be found in the ſaid tables (many compound numbers having been omitted, to reduce them into ſo narrow a compaſs) you muſt add the parts of ſuch two or three numbers together, as will compoſe it, and the total will be the decimal ſought. Thus, if the decimal of 17 pennyweights, the integer of a pound *Troy*, were to be ſought in the 2d table ,04166 the decimal of 10 pennyweights, and ,029166 the decimal of 7 pennyweights, being added together, the total ,070832 will be found the anſwer; or, if in table the fourth, you would find the decimal of 352 days,

add	,821918	*the*	300	
	,136986	*Decimals*	50	*Days.*
	,005479	*of*	2	
total	,964318	*anſwering*	352	*Days.*

and ſo of any other.

Decimal Tables of Coin & Weight

Table 1st

* Coin 1£ ster: ye Integer

S	Dec:	S	Dec:
19	,95	9	,45
18	,9	8	,4
17	,85	7	,35
16	,8	6	,3
15	,75	5	,25
14	,7	4	,2
13	,65	3	,15
12	,6	2	,1
11	,55	1	,05
10	,5		

Pence	Decim:
11	,045833
10	,041666
9	,0375
8	,033333
7	,029166
6	,025
5	,020833
4	,016666
3	,0125
2	,008333
1	,004166

Farthings	Decim:
3	,003125
2	,002083
1	,001042

Table 2d

Troy Weight 1 lb. the Integer

Ounces	Decim:
11	,916666
10	,833333
9	,75
8	,666666
7	,583333
6	,5
5	,416666
4	,333333
3	,25
2	,166666
1	,083333

Note. This Table of Oz: will also serve for Inches Months or Doz:

Penny wts	Decim:
10	,041666
9	,0375
8	,033333
7	,029166
6	,025
5	,020833
4	,016666
3	,0125
2	,008333
1	,004166

Grains	Decim:
20	,003472
10	,001736
9	,001562
8	,001389
7	,001215
6	,001042
5	,000868
4	,000694
3	,000521
2	,000347
1	,000173
½	,000086

Table 3d

Avoirdupoise 112 lb. ye Integer.

Quarters	Decim:
3	,75
2	,5
1	,25

Pounds	Decim:
20	,178571
10	,089286
9	,080357
8	,071428
7	,0625
6	,053571
5	,044643
4	,035714
3	,026786
2	,017857
1	,008928

Ounces.	Decim:
10	,005580
9	,005022
8	,004464
7	,003906
6	,003348
5	,002790
4	,002232
3	,001673
2	,001116
1	,000558

Drams	Decim:
10	,000348
9	,000313
8	,000279
7	,000244
6	,000209
5	,000174
4	,000139
3	,000104
2	,000069
1	,000034
½	,000017

Webster scripsit. Bickham Sculpsit.

Between Pages [illegible]

Decimal Tables of Measure and Time

Table 4th

Time 1 year ye Integer

Days	Decim:
300	,821918
200	,547945
100	,273973
90	,246575
80	,219178
70	,191781
60	,164383
50	,136986
40	,109589
30	,082192
20	,054794
10	,027397
9	,024657
8	,021918
7	,019178
6	,016438
5	,013699
4	,010959
3	,008219
2	,005479
1	,002739

Table 5th

Measure Liquid. Dry. Integer 1 Gall: 1 Quarter

Pints	Decim:	Bush:
7	,875	7
6	,75	6
5	,625	5
4	,5	4
3	,375	3
2	,25	2
1	,125	1

qr. Pts	Decim:	Pecks
3	,09375	3
2	,0625	2
1	,03125	1

Decim:	qrs. Pecks
,023437	3
,015625	2
,007812	1

Decim:	Pints
,005859	3
,003906	2
,001953	1

Cloth Measure 1 Yard ye Integer

Quarters	Decim:
3	,75
2	,5
1	,25

Nails	Decim:
3	,1875
2	,125
1	,0625

Table 6th

Time 1 Day ye Integer

Hours	Decim:
20	,833333
10	,416666
9	,375
8	,333333
7	,291666
6	,25
5	,208333
4	,166666
3	,125
2	,083333
1	,041666

Minutes	Decim:
50	,034722
40	,027777
30	,020833
20	,013888
10	,006944
9	,00625
8	,005555
7	,004861
6	,004166
5	,003472
4	,002777
3	,002083
2	,001388
1	,000694

Webster scrip.t

Bickham scul.t

CHAP. III.

Of the RULE *of* THREE.

THIS *Rule of* PROPORTION having been already ſufficiently explain'd, both as to its nature and operation in *Vulgar Arithmetick*, 'tis needleſs to repeat here what has been there ſaid. Only therefore obſerve, that inſtead of preparing your numbers, by reducing them into their loweſt denomination, you muſt now bring their fractional parts into decimals. See the examples.

EXAMPLES.

Ex. 1. If the price of 26 yards and $\frac{1}{2}$ of drugget is 3 *l.* 16 *s.* 3 *d.* what will 32 yards and $\frac{1}{4}$ come to?

The fractional parts of the numbers being reduced to decimals, and the queſtion ſtated as uſual, the work will ſtand as follows:

Yards. *l.* *Yards.*
If 26,5—3,8125—32,25
32,25

190625
76250
76250
114375

26,5)122,953125(4,63974
1695
1053
2581
1962
1075
15

The quotient valu'd by the ſhort rule for money, makes 4 l. 12 s. 9 d ½.

Ex. 2. What will the pay of 540 men amount to at 1 *l.* 5 *s.* 6 *d. per* man?

Man. *l.* *Men.*
If 1———1,275———540
540

51000
6375

Anſwer, 688,500 *or* 688 *l.* 10 *s.*

For more examples, take thoſe given in *Vulgar Arithmetick.*

The Indirect RULE *of* THREE.

THE Indirect RULE of THREE is alſo the ſame here as in *Vulgar Arithmetick*, differing only in the preparation of the numbers.

EXAMPLE.

What muſt the penny-loaf weigh, when wheat is 10 *s.* the buſhel, if, when at 6 *s.* 8 *d. per* buſhel the penny-loaf weighs 5 *oz.* ½?

If ,33333——5,5——,5
5,5

166665
166665

,5)1,833315(

,366663

	oz.	*dwts.*	*grs.*
Anſw.	3	13	7

Take alſo one Example in the DOUBLE RULE.

If the carriage of ½ a C. *wt.* 40 miles, comes to 6*d.* what will be the charge of carrying 16 C. ¼ 1000 miles?

First, *If* ,5——,025——16,25

,025

8125

3250

,5),40625(

Miles.

Then, *If* 40—— ,8125——100

100

40)81,2500(

facit 2,03125

or,

2 *l.* 0 *s.* 7 *d* ½.

CHAP IV.

Of PRACTICE.

SOME queſtions in this rule may alſo be advantagiouſly work'd by decimals; as,

1*ſt.* When the given price is juſt two ſhillings, 'tis only ſeparating the right hand figure for a decimal, the reſt are pounds; ſo in the

EXAMPLE.

5295 ℔, at 2 *s. per* ℔, *comes to* $\begin{cases} 529{,}5 \\ \text{or} \\ 529\ l.\ 10\ s. \end{cases}$

2*d.* When the price is any *aliquot* part of 2 *s* 'tis only ſeparating the right hand figure, as before, and then dividing the number by ſuch part.

EXAMPLE.

oz. *d.*
258,3 at 8 *per oz.*
l. *s.*
,8 *d.* ⅓ 86,1 *facit*, *or* 86 2

3*d.* When the given price can be divided into *aliquot* parts of two ſhillings, ſeparate as before, and take ſuch parts.

E X A M P L E.

℔. d.
d. 563,8 at 9 *per* ℔.

6 ¼ 140,95
3 ½ 70,475

facit 211,425 *or* 211 *l.* 8 *s.* 6 *d.*

4th. When the price is any number of ſhillings whatſoever, 'tis only multiplying by their half, (which is the decimal of them) and the product is the anſwer.

E X A M P L E S.

℔. *s.*
7296 *at* 12 *s. per* ℔.
,6

facit 4377,6 *or* 4377 *l.* 12 *s.*

oz. *s.*
8264 *at* 15 *s. per oz.*
,75

41320
57848

*l.*6198,00 *facit.*

5th. The requir'd price of any quantity may be found, by multiplying the ſaid quantity by the decimal of the price given.

E X A M P L E.

℔. *l.* *d.*
8397 *at* 5 13 6 *per* ℔.
5,675

41985
58779
50382
41985

facit 47652,975 *or* 47652 *l.* 19 *s.* 6 *d.*

6th. If there are fractional parts in the given quantity, they muſt be reduced to decimals, as well as the fractional parts of the price, as in the following

EXAMPLES.

℔. s.
368 ½ *at* 13 *per* ℔.
368,5
,65
18425
22110
facit 239,525
or,
239 *l.* 10 *s.* 6 *d.*

C. *qrs.* ℔. *l. s. d.*
7368 3 14 *at* 1 12 6
7368,875
1,625
36844375
14737750
44213250
7368875
facit 11974,421875
or,
11974 *l.* 8 *s.* 5 *d*¼.

The queſtions propos'd in *Vulgar Arithmetick*, will here alſo ſerve for more examples.

CHAP. V.

Of INTEREST *and* REBATE.

THESE rules having been already ſufficiently explain'd in *Vulgar Arithmetick*, it only now remains to ſhew how much more advantagiouſly they may be work'd by decimals. And *firſt*,

SIMPLE INTEREST.

1ſt. THE annual intereſt of any ſum of money is found by only multiplying the given principal by the intereſt of one pound for a year, which

$$\text{at}\left\{\begin{matrix}4\\5\\6\end{matrix}\right\}\textit{per Cent.}\text{ is}\left\{\begin{matrix},04\\,05\\,06\end{matrix}\right.$$

found by dividing the rate of intereſt by 100.

EXAMPLES.

l. *l.*
1*ſt*. What's a year's intereſt of 536 at 5 *per Cent*?
,05

facit 26,80 *or* 26 *l.* 16 *s.*

2*d.* What will the intereſt of 2367 *l.* come to in a year, at 6 *l. per Cent. per Ann.*? ,06

facit 142,02 *or* 142 *l.* 0 *s.* 5 *d.*

2*d.* Thus a year's intereſt of any ſum being found, 'tis only multiplying that ſum by 2, 3, 4, *&c.* and the product is the intereſt of the ſaid ſum for 2, 3, 4, or more years:

EXAMPLE.

What's the intereſt of 623 for 3 years, at 4 *per Cent.*?
,04

24,92 *Intereſt for* 1 *year.*
3

74,76
or, } *Intereſt for* 3 *years.*
74 *l.* 15 *s.* 2 *d* $\frac{1}{2}$.

3*d.* Again, If the time requir'd be months, 'tis only dividing the year's amount by ſuch parts as the given months are of a year.

EXAMPLES.

Ex. 1*ſt.* What is the intereſt of 539 *l.* for four months, at 5 *l. per Cent. per Annum?*

l.
539
,05

26,95 *Intereſt for a year.*

Months
$4\frac{1}{3}$ 8,9833 } *facit.*
or,
8 *l.* 19 *s.* 8 *d.*

Ex. 2*d.* What will the intereſt of 729 *l.* 10 *s.* 6 *d.* come to in a year and 5 months, at 6 *per Cent. per Annum?*

Note, If there are any odd ſhillings, pence, and farthings in the principal ſum, they muſt be reduced to decimals, as in this example.

729,525
,06

Months 43,77150 *Intereſt for a year.*
$4\frac{1}{3}$ 14,59050 *Intereſt for* 4 *months.*
$1\frac{1}{4}$ 3,647625 *Intereſt for* 1 *month.*

facit 62,009625 *Total Intereſt.*
or,
62 *l.* 0 *s.* 2 *d* $\frac{1}{4}$.

4*th.* But if it be required to find the intereſt of a ſum of money for any number of days, you muſt either multiply the year's amount by the number of days propos'd, and divide the product by 365, or (which will be ſometimes ſhorter) divide the year's amount by 365, and multiply the quotient by the number of days propos'd.

EXAMPLE.

What intereſt will 563 *l.* amount to in 126 days, at 6 *l. per Cent. per Annum.*

l.
563
,06

33,78
126

20268
6756
3378

365)4256,28(11,661

·606

2412

2228

· · 380

·15

365)33,78000(,09254

· · 930

2000

1750

290
,09254
126

55524
18508
9254

11,66004

But for the more ready caſting up the intereſt of money for days, it will be neceſſary to have the intereſt of 1 *l.* for 1 day, at all rates ready calculated as a table or ſtandard. Thus:

The INTEREST *of* 1 *l. for* 1 *Day.*

At	1	*per Cent.* is	,00002739726
	2		,00005479452
	3		,00008219178
	4		,00010958904
	5		,00013698630
	6		,00016438356
	7		,00019178082
	8		,00021917808
	9		,00024657534
	10		.00027397260

The inrereſt of 1 *l.* for one day, is thus found; the given rate or intereſt of 100 *l.* for a year, being divided by 100, quotes the intereſt of 1 *l.* for a year; which again divided by 365, gives the required intereſt of 1 *l.* for 1 day; which, when multiply'd into both the number of days, and the principal ſum, produces the intereſt for the time required.

EXAMPLE.

What's the intereſt of 560 *l.* for 60 days, at 5 *l.* *per Cent. per Annum?*

,0001369863
560 *Principal.*

82191780
6849315

767123280
60 *Days.*

Facit 4,6027396800 *or* 4 *l.* 12 *s.* 0 *d* ½.

The

To front Page 193.

A Table

Shewing the Number of Days from any day in any Month to the same day in any other Month.

From	Jan:	Feb:	Mar:	April.	May.	June.
To	Feb:31	Mar:28	April31.	May.30	June31	July.30
	Mar:59	April59	May.61	June.61	July.61	Aug:61
	April90	May.89	June92	July.91	Aug:92	Sep:92
	May120	June120	July122	Aug:122	Sep:123	Oct:122
	June151	July.150	Aug:153	Sep:153	Oct:153	Nov:153
	July.181	Aug:181	Sep:184	Oct: 183	Nov:184	Dec:183
	Aug:212	Sep:212	Oct:214	Nov:214	Dec:214	Jan:214
	Sep:243	Oct:242	Nov:245	Dec:244	Jan:245	Feb:245
	Oct: 273	Nov:273	Dec:275	Jan:275	Feb:276	Mar:273
	Nov:304	Dec:303	Jan:306	Feb:306	Mar:304	April304
	Dec:334	Jan:334	Feb:337	Mar:334	April335	May334
	Jan:365	Feb:365	Mar:365	April365	May365	June365

From	July.	Aug:	Sep:	Oct:	Nov:	Dec:
To	Aug:31	Sep:31	Oct:30	Nov:31	Dec:30	Jan:31
	Sep:62	Oct:61	Nov:61	Dec:61	Jan:61	Feb:62
	Oct:92	Nov:92	Dec:91	Jan:92	Feb:92	Mar:90
	Nov:123	Dec:122	Jan:122	Feb:123	Mar:120	April121
	Dec:153	Jan:153	Feb:153	Mar:161	April151	May151
	Jan:184	Feb:184	Mar:181	April182	May:181	June.182
	Feb:215	Mar:212	April212	May212	June.212	July.212
	Mar:243	April243	May.242	June243	July242	Aug:243
	April.274	May.273	June273	July273	Aug:273	Sep:274
	May.304	June304	July303	Aug:304	Sep:304	Oct:304
	June.335	July334	Aug:334	Sep:335	Oct:334	Nov:335
	July.365	Aug:365	Sep:365	Oct:365	Nov:365	Dec:365

Webster Script.
Bickham Sculp.

The year being divided into 12 unequal months, the following verſes may ſerve as a *memorandum* of the number of days in each, *viz.*

Thirty days hath September,
April, June, *and* November;
February *hath twenty eight alone*,
And all the reſt have thirty one.

But without any farther trouble, the oppoſite table ſhews, by inſpection only, the number of days from any day in any month, to the ſame day in any other month. As ſuppoſe the number of days, betwixt the 4th of *February* and the 4th of *September*, were requir'd; looking in the column under *February* for *September*, againſt that month will ſtand, 212, the number of days betwixt the ſaid times; ſo again, from the 8th of *June* to the 8th of *October*, appears by the table to be 122 days, that being the number againſt *October*, in the column under *June*.

Note, If the given days are different, 'tis only adding, or ſubſtracting their inequality to, or from the tabular number. Thus had the firſt example been from the 4th of *February*, to the 8th of *September*, it had been 4 days more than 212, *viz.* 216; or had the time, in the laſt example, been from the 8th of *June*, to the 4th of *October*, it had been 4 days leſs than 122, *viz.* 118.

Note alſo, If the time exceeds a year, 365 days muſt be added. Thus from the 4th of *February* 1714, to the 4th of *September* 1715, will be found to be 577 days, the ſum of 212, and 365. See the Table.

COMPOUND

COMPOUND INTEREST.

1ſt. TO find the amount of any ſum, at any rate of compound intereſt, for any number of years.

RULE.

Multiply the rate, that is, the amount of 1 *l.* for a year, which at 6 *per Cent.* is 1,06, at 5 *per Cent.* 1,05, *&c.*) ſo often into it ſelf, as are the number of years propos'd, wanting one; and the laſt product multiply'd by the principal, will give the amount required.

EXAMPLE.

What's the amount of 500 *l.* forborn four years, at 4 *l. per Cent. per Annum?*

1,04
1,04

1,0816
1,04

1,124864
1,04

1,16985856
500

facit 584,92928000

And thus is the firſt intereſt-table (which ſhews the amount of 1 *l.* for any number of yeats, under 33, at the rates of 5 and 6 *l. per Cent. per Annum*) form'd.

The uſe of which is plain and eaſy; for multiplying the figures ſtanding againſt the number of years requir'd; and under the given rate, by the propos'd principal, the product is the amount deſired.

Thus, if the amount of 40 *l.* in 20 years, at 5 *l.* *per Cent. per Annum*, were requir'd, 2,65329 the tabular number, multiply'd by 40 *l.* the principal ſum, gives 106,13160, or 106 *l.* 2 *s.* 7 *d* ½, the *Anſwer*. And ſo any other.

2*d.* To find the amount of any annuity, or yearly penſion, forborn any number of years whatſoever, at any rate of compound intereſt.

RULE.

Multiply the firſt yearly payment by the rate, and to the product add the ſecond yearly payment, the ſum is the amount in 2 years; which, multiply'd again by the rate, the product, with the addition of the third yearly payment, is the amount for 3 years, *&c.*

EXAMPLE.

What will a penſion of 30 *l.* *per Annum* amount to, being forborn 4 years, at 5 *l.* *per Cent. per Annum*?

	l.	
Firſt yearly payment	30	
multiply'd by the rate	1,05	
	31,50	
Second yearly payment added	30	
gives	61,50	*the amount in 2 years.*
	1,05	
	64,5750	
Third yearly payment	30	
	94,5750	*Amount in 3 years.*
	1,05	
	99,303750	
Fourth yearly payment	30	
	129,303750	*Amount in 4 years.*

And after this manner is the ſecond intereſt table (which ſhews the amount of 1 *l.* annuity for any number of years under 33, at the rates of 5 and 6 *l. per Cent. per Annum*, compound intereſt) compos'd.

But this ſecond table may more eaſily be form'd from the firſt, thus: the firſt line of this ſecond table, or firſt year's amount being known, add to it the firſt line of the firſt table, the ſum is the amount for 2 years, or the ſecond line of this table; to which again adding the ſecond line of the firſt table, you have the third of this, or the amount for three years, &c.

EXAMPLE at 6 l. per Cent.

l.
1, —— *Firſt year of the ſecond table.*
1,06 —— *Firſt of the firſt table.*

2,06 —— *Second year of the ſecond table.*
1,1236 —— *Second year of the firſt.*

3,1836 —— *Third year of the ſecond.*
1,19101 —— *Third year of the firſt.*

4,37461 —— *Fourth year of the ſecond,* &c.

The uſe of this ſecond table is in the ſame manner as the firſt, as ſuppoſe the foregoing queſtion for an example.

EXAMPLE.

4,31012 { *the amount of* 1 *l. annuity for* 4 *years, at* 5 *l.* per Cent.
multiply'd by 30 *the yearly ſum,*

gives 129,30360 *as before.*

RE-

REBATE *or* DISCOMPT.

REBATE, according to ſimple intereſt, having been ſufficiently explain'd in *Vulgar Arithmetick*, I ſhall only here ſet down one example work'd decimally the common way. And, to compleat the ſection, add Mr. *Hatton*'s new method of finding the preſent worth and diſcompt of money, as deliver'd in his *Syſtem of Arithmetick*, pag. 188.

EXAMPLE.

What diſcompt muſt be allow'd on a bill of 500 *l.* paid 20 days before it is due, rebate at 5 *l. per Cent. per Annum?* And what preſent money muſt be paid?

The common way.

```
    Days.      l.      Days.
 If 365 ——— 5 ——— 20
                      5
                  ————
          365) 1000(27397
                 ————
                  2700
                  ————
                   1450
                   ————
                    3550
                    ————
                     2650
                     ————
                       95
```

Then for the diſcompt:

If 100,27397——,27397——500

500

100,27397)136,98500(1,3661 *diſcompt.*

36711030

66288390

61240080

10756980

·629593

or for the preſent worth,

If 100,27397—,100—500

500

100,27397)50000,000(498,6338 *preſent worth.*

98904120

86575470

63562940

33985580

39033890

89516990

9297814

Mr. HATTON's *New Method.*

1. *For the preſent worth,* multiply the days in a year, the principal given, and 100 into each other, for a dividend; and add the product of 365 by 100, to that of the days multiply'd into the rate given for a diviſor, ſo the quotient ariſing is the anſwer.

Thus in the foregoing example $365 \times 500 \times 100 = 18250000$ for the dividend, and $365 \times 100 + 20 \times 5 = 36600$ for the diviſor.

See the work.

366|00)182500|00(498,6338 *preſent worth.*

3610

3160

2320

1240

1420

322

2. ***For the diſcompt***, multiply the rate principal, and days given together for a dividend; and proceeding as before directed for a diviſor, the quotient will be the anſwer.

Thus keeping ſtill the ſame example, $5 \times 500 \times 20 = 50000$ for the dividend, and $365 \times 100 + 20 \times 5 = 36600$ for a diviſor, as before.

See the work.

366|00)500|00(1,3661 *diſcompt.*

1340

2420

2240

440

74

The truth of both theſe methods appears, not only by their agreement, but alſo by the addition of the preſent worth, and diſcompt, found, which, as near as can be, makes the principal.

Thus 498,6338 *the preſent worth,*
with 1,3661 *the diſcompt,*

makes 499,9999 = 500 *l. the given principal.*

REBATE at *COMPOUND INTEREST.*

1*ſt.* To find the preſent worth of any ſum to be paid at any number of years to come, rebate being made at any rate of compound intereſt.

RULE.

Divide the principal continually ſo many times by the rate, as are the number of years propoſ'd, and the laſt quotient is the anſwer.

EXAMPLE.

What is the preſent worth of 1 *l.* to be paid at the end of 6 years, rebate at 6 *l. per Cent. per Ann.* compound intereſt?

	l.				*years*
The principal	1,	*divi-*	,943396	*preſent worth at the end of*	1
	,943396	*ded by*	,889997		2
	,889997	1,06	,839619		3
	,839619	*the*	,792094		4
	,792094	*rate,*	,747258		5
	,747258	*gives*	,704960		6

And thus is the third table made, which ſhews the preſent worth of 1 *l.* due at any number of years to come, under 33, rebate at 5 and 6 *l. per Cent. per Annum,* compound intereſt; the uſe of which is, as the other tables, only by multiplying the preſent worth of 1 *l.* by the given principal, and the product is the preſent worth required.

Thus ſuppoſe in the example above, the principal had been 200 *l.* the preſent worth of 1 *l.* *viz.* ,704960 multiply'd by 200, gives 140,992000, the preſent worth of 200 *l.* and ſo any other.

2*d.* To find the preſent worth of any annuity, or yearly penſion, to continue any number of years; rebate being made at any rate, *per Cent. per Annum*, compound intereſt.

RULE.

Find by the foregoing rule the preſent worth of the propos'd annuity, for 1, 2, 3, or ſo many years as are demanded; and the ſum of thoſe reſpective preſent worths will be the value of the annuity; that is, the firſt and ſecond of thoſe values, added together, will be the preſent worth for two years; and the firſt, ſecond, and third, added, for three years, *&c.*

EXAMPLE.

What's the preſent worth of an annuity of 50 *l.* to continue four years, rebate at 5 *l.* *per Cent. per Annum?*

l.		
50,	*divided by* 1,05 *the rate, gives*	47,61904
47,61904		45,35147
45,35147		43,19187
43,19187		41,13512
The total of which is		177,29750

the preſent (worth requir'd.

And after this manner is made table the fourth, which ſhews the preſent worth of 1 *l.* annuity, to continue any number of years under 33, rebate at 5 and 6 *l. per Cent. per Annum*, compound intereſt. Which table, as the others, will give the preſent worth of any ſum whatſoever, by only multiplying the value of 1 *l.* by the ſum propos'd. Thus ſuppoſe the foregoing example.

EXAMPLE.

3,54595 *the preſent worth of* 1 *l. An. for* 4 *yrs.*
multiply'd by 50 *the Annuity propos'd.*

gives 177,29750 *as before.*

The fourth table may be alſo form'd from the third, thus;

The firſt year is the ſame in both; the firſt and ſecond years of the third, added together, make the ſecond of the fourth; the ſecond year of the fourth, added to the third year of the third, makes the third of the fourth table, *&c.* See the example.

EXAMPLE at 6 l. *per Cent.*

,94339 * *The firſt year of both tables.*
,88999 *The ſecond year of the third table.*

1,83339 *The ſecond year of the fourth.*
,83961 *The third year of the third.*

2,67300 *The third year of the fourth,* &c.

---

* *Note*, 1 is added to the right hand figure, in conſideration of the places omitted.

Decimal Tables

shewing

TABLE 1st. The Amount of one Pound for any Number of Years under 33, at ye Rates of 5&6 p Ct. p An: Com: Inter:

TABLE 2d. The Amount of 1£ Annuity for any number of Years under 33, at ye Rates of 5&6 p Ct. p An: Com: Interest.

Rates		Years	Rates	
5	6		5	6
1,05000	1,06000	1	1,00000	1,00000
1,10250	1,12360	2	2,05000	2,06000
1,15762	1,19101	3	3,15250	3,18360
1,21550	1,26247	4	4,31012	4,37461
1,27628	1,33822	5	5,52563	5,63709
1,34009	1,41852	6	6,80191	6,97532
1,40710	1,50363	7	8,14200	8,39383
1.47745	1,59384	8	9.54910	9,89746
1,55132	1,68948	9	11,02656	11,49131
1,62889	1,79084	10	12,57789	13,18079
1,71034	1,89829	11	14,20678	14,97164
1,79585	2,01219	12	15,91712	16,86994
1,88565	2,13292	13	17,71298	18,88213
1,97993	2,26090	14	19,59863	21,01506
2,07892	2,39655	15	21,57856	23,27597
2,18287	2,54035	16	23,65749	25,67252
2,29201	2,69277	17	25,84036	28,21288
2,40662	2,85434	18	28,13238	30,90565
2,52695	3,02559	19	30,53900	33,75999
2,65329	3,20713	20	33,06595	36,78559
2,78596	3,39956	21	35,71925	39,99272
2,92526	3,60353	22	38,50521	43,39229
3,07152	3,81975	23	41,43047	46,99582
3,22510	4,04893	24	44,50199	50,81557
3,38635	4,29187	25	47,72709	54,86451
3,55567	4,54938	26	51,11345	59,15638
3,73345	4,82234	27	54,66912	63,70576
3,92013	5,11168	28	58,40258	68,52811
4,11613	5,41838	29	62,32271	73,63979
4,32194	5,74349	30	66,43884	79,05818
4,53804	6,08810	31	70,76079	84,80167
4,76494	6,45338	32	75,29883	90,88977

Webster Scripsit. Bickham Sculpsit.

Between Pages 202 & 203.

Decimal Tables

shewing

TABLE 3^{d} The present Worth of 1£ due at any number of years to come under 33, Rebate at 5 & 6 ℘ C^{t} ℘ An: Com: Interest

TABLE 4th The present Worth of 1£ Annuity to continue any number of Years under 33, Rebate at 5 & 6 ℘ C^{t} ℘ An: Com: Interest

Rates		Years	Rates	
5	6		5	6
,952381	,943396	1	0,95238	0,94339
,907030	,889996	2	1,85941	1,83339
,863838	,839619	3	2,72324	2.67301
,822702	,792093	4	3,54595	3,46510
,783526	,747258	5	4,32947	4,21236
,746215	,704960	6	5,07569	4,91732
,710682	,665057	7	5,78637	5,58238
,676839	,627412	8	6,46321	6,20979
,644609	,591898	9	7,10782	6,80169
,613913	,558394	10	7,72173	7,36008
,584679	,526787	11	8,30641	7,88687
,556837	,496969	12	8,863325	8,38384
,530321	,468839	13	9,39357	8,85268
,505068	,442301	14	9,89864	9,29498
,481017	,417265	15	10,37965	9,71225
,458111	,393647	16	10,83777	10,10589
,436296	,371364	17	11,27406	10,47726
,415520	,350343	18	11,68958	10,82760
,395734	,330513	19	12,08532	11,15811
,376889	,311804	20	12,46221	11,46992
,358942	,294155	21	12,82115	11,76407
,341849	,277505	22	13,16300	12,04158
,325571	,261797	23	13,48857	12,30338
,310067	,246978	24	13,79864	12,55035
,295302	,232998	25	14,09394	12,78335
,281240	,219810	26	14,37518	13,00316
,267848	,207368	27	14,64303	13,21053
,255093	,195630	28	14,89812	13,40616
,242946	,184556	29	15,14107	13,59072
,231377	,174110	30	15,37245	13,76483
,220359	,164254	31	15,59281	13,92908
,209865	,154956	32	15,80267	14,08404

Webster scrip.^{t} Bickham sculp.^{t}

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More EXAMPLES *for the Exerciſe of the foregoing* RULES *and* TABLES.

Ex. 1*ſt*. What will 1000 *l.* increaſe to, forborn 22 years, at 5 *per Cent. per Annum* compound intereſt?

Anſwer, 2925 *l.* 5 *s.* 2 *d* ½.

Ex. 2*d*. What would an annuity of 520 *l.* forborn 17 years, at 6 *per Cent.* compound intereſt, amount to?

Anſwer, 14670 *l.* 13 *s.* 11 *d* ¼.

Ex. 3*d*. What preſent money would diſcharge a debt of 7500 *l.* to be paid at the end of 10 years, rebate being made at 5 *l. per Cent. per Annum*?

Anſwer, 4604 *l.* 6 *s.* 11 *d* ¼.

Ex. 4*th*. What is a yearly rent of 65 *l.* to continue 30 years, worth in ready money, rebate being made at 6 *per Cent. per Annum* compound intereſt?

Anſwer, 894 *l.* 14 *s.* 3 *d* ¼.

Ex. 5*th*. What is the preſent worth of a reverſion of a leaſe of 500 *l. per Annum*, to continue 20 years, but not to commence 'till after the end of five years, allowing the purchaſer 6 *per Cent.* compound intereſt?

Note, To work queſtions of this nature, there muſt be two diſtinct operations, *viz*.

1*ſt*. The preſent worth of the propos'd annuity, for the given time of its continuance, muſt be found as if it were immediately to commence. Then,

2*d*. See what principal, forborn at the given intereſt, would, in the time of the reverſion, amount to the aforeſaid preſent worth, and that principal will be the preſent worth of ſuch annuity in reverſion.

Thus by table the 4*th, the preſent worth of* 1 l. per Annum *to continue* 20 *years, at* 6 l. per Cent. *is found to be*

11,46992

which multiply'd by the propos'd annuity, 500

gives 5734,9600

Then in the firſt table againſt 5 years, the time of the reverſion, under the ſame rate of intereſt is found 1,33822 the amount of 1 *l.* in the ſaid time; therefore

If 1,33822 *ariſe from* 1 *l. from what principal does* 5734,96000 *ariſe?*

Anſwer, 4285,5135, *or* 4285 *l.* 10 *s.* 3 *d* ¼.

Or it may more eaſily be done by firſt finding the preſent worth of the annuity for the whole time both of poſſeſſion and reverſion, and then deducting the preſent worth of the poſſeſſion; the remainder will be the preſent worth of the reverſion. Thus 12,78335, the tabular number, againſt 25 years, the whole time being multiply'd by 500, gives 6391,67500, from which ſubſtracting 2106,18000 the produce of 4,21236 the tabular number againſt 5 years, the time of the poſſeſſion, the remainder 4285,49500 is the value of the reverſion: Within a trifle the ſame anſwer as before.

FREEHOLD ESTATES.

TO find the preſent worth of any annual rent, to continue for ever, commonly call'd fee-ſimple.

RULE.

R U L E.

Divide the propos'd rent by the intereſt of 1 *l.* for one year, (which at 5 *l. per Cent.* is ,05, or at 6 *per Cent.* ,06, as before ſhewn) and the quotient is the preſent value of the eſtate.

E X A M P L E.

What is an eſtate of 200 *l. per Annum*, to continue for ever, worth in ready money, allowing the purchaſer 5 *l. per Cent. per Annum* compound intereſt?

,05)200,00(

Anſwer, 4000 *l.*

Or it may be done by multiplying the fee-ſimple of 1 *l.* by the yearly rent propos'd.

The fee-ſimple of 1 *l. per Annum* compound intereſt, at

1 *per Cent.*	2 *per Cent.*	3 *per Cent.*	4 *per Cent.*	5 *per Cent.*
100,00000	50,0000	33,33333	25,00000	20,00000

6 *per Cent.*	7 *per Cent.*	8 *per Cent.*	9 *per Cent.*	10 *per Cent.*
16,66667*	14,28571	12,50000	11,11111	10,00000

* *Note*, In contracting a line of decimals from many to fewer places, 1 is added to the laſt figure retain'd, if the next omitted figure exceedeth 5; which rule hath been all along obſerv'd in the tables, &c.

The foregoing queſtion work'd by the laſt rule.

l.
The fee-ſimple *of* 1 *l. per Ann. at* 5 *l. per Cent. is* 20
Which multiply'd by the yearly rent, 200
gives as before, 4000

I will now only add another example or two, for the reader's exerciſe, and conclude this rule.

1. *A* has the poſſeſſion of an eſtate of 130 *l per Annum*, to continue 20 years; *B* has the reverſion of the ſame, from that time for ever. What muſt *A* give *B*, if he would purchaſe his reverſion? And what muſt *B* give *A*, if he would buy his poſſeſſion, accounting 6 *per Cent.* compound intereſt in each caſe?

		l.	*s.*	*d.*
Anſwer,	A'*s poſſeſſion is worth*	1491	1	9 ¼
	B's *reverſion is worth*	675	11	6 ½

2. *A* agrees with *B* for an annuity of 600 *l. per Annum*, to continue 25 years, to give him the preſent worth of it, at 6 *l. per Cent. per Annum*; but not having money enough by him, offers to make over to him a freehold eſtate of 12 *l. per Annum* at the ſame intereſt, What money beſides will pay his purchaſe?

Anſwer, 7470*l.* 0*s.* 2*d* ½.

CHAP.

CHAP. VI.

Of FELLOWSHIP.

WIthout taking any notice of the common way of working this rule, obſerve this much more compendious method.

RULE.

Divide the whole gain, or loſs, by the whole ſtock, and multiply the quotient by each man's particular ſtock, the ſeveral products are the reſpective gains of each.

Note, It is neceſſary to place ſo many cyphers on the right hand of your dividend, as will bring out ſix or ſeven places of decimals in the quotient.

EXAMPLE.

Suppoſe *A*, *B*, and *C* trading together, *A*. puts in 500 *l*. *B* 700 *l*. and *C*. 1200 *l*. the whole gain is 836 *l*. What ſhare of it belongs to each?

```
       l.
  A    500
  B    700
  C   1200
      ----
Total ſtock 2400)836,000000(,348333
                 ----------
                  11600
                  -----
                   20000
                   -----
                    8000
                   -----
                     8000
                    -----
                      8000
                     -----
                       800

     ,348333          ,348333         ,348333
         500              700            1200
    ----------       ----------      ----------
A's ſhare 174,166500 B's 243,833100 C's 417,999600
B's ſhare 243,833100
C's ſhare 417,999600
          ----------
Tot. gain 835,999200, or 836 l. as near as can be.
```

See another EXAMPLE *with Time.*

A puts into company 525 *l.* 10 *s.* for ſix months, B 382 *l.* 15 *s.* for eight months, and C. 1000 *l.* for four months; they gain in all 1286 *l.* 12 *s.* How much is that for each?

```
              525,5            382,75            1000
                  6                 8               4
              ------           ------            ----
A's ſtock    3153,0           3062,00            4000
B's ſtock    3062
C's ſtock    4000
             ------
     10215)1286,600000(,125952

             ,125952           ,125952          ,125952
                3153              3062             4000
           ----------       ----------       ----------
A's gain  397,126656        385,665024       503,808000
B's——     385,665024
C's——     503,808000
         -----------
```

Total 1286,599680 *or* 1286 *l.* 12 *s. as near as can be.*

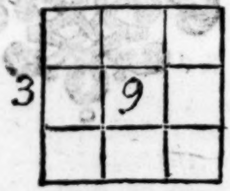
CHAP. VII.

Of EVOLUTION, *or* EXTRACTION *of* ROOTS.

The SQUARE ROOT.

EXTRACTION of the SQUARE ROOT is finding the ſide of a ſquare figure; or, numerically ſpeaking, it is finding what number, multiply'd by it ſelf, will produce the number given. Thus the ſquare root of 16 is 4, 4 times 4 making 16.

What a ſquare is, may be ſeen in the following figure, which being divided every way into 3 equal parts, its whole content or ſquare is 9, and its ſide or root is 3.



Square Numbers are either ſingle or compound.

A ſingle ſquare number is always leſs than 100, being produc'd by the multiplication of ſome one ſingle figure by it ſelf, as 16 from 4, *&c.* So that the root of any ſingle ſquare may be found in the annex'd table, always taking the root of the next leſs ſquare, for any number not there inſerted, as for 26 take 5, or 3 for 10, *&c.*

Squares	1	4	9	16	25	36	49	64	81
Roots	1	2	3	4	5	6	7	8	9

A compound ſquare number being compos'd by the multiplication of two or more figures by themſelves, always exceeds 100; as 144, which is 12 times 12; or 195, which is 15 times 15, *&c.*

If the root therefore is expreſs'd by two figures, its ſquare muſt at laſt conſiſt of three, for the leaſt root expreſs'd by two figures is 10, whoſe ſquare is 100. And if the root has three figures, its ſquare muſt at leaſt have five. If four, the ſquare has ſeven, *&c.* ſo that you cannot augment the root one figure, but you encreaſe the ſquare two.

To find the root of any compound ſquare number, as ſuppoſe 2704,

1*ſt*. You muſt diſtinguiſh it into ſingle ſquares, by placing dots over every other figure, beginning at the right hand, thus,

$$2\dot{7}0\dot{4}$$

And ſo many dots as happen, ſo many places will the root conſiſt of, which in this example are two.

2d'y. Drawing a crooked line on the right hand of your number, as in diviſion, find the root of your firſt ſingle ſquare, and place it in the quotient. See the work:

· ·
2704(5

3dly. Placing the ſquare of the root found, under the firſt ſingle ſquare, ſubſtract, and ſet down the remainder; bringing down to it the next ſingle ſquare, call the line a *Reſolvend.*

· ·
2704(5
25
———
· 204 *Reſolvend.*

4thly. Drawing another crooked line on the left hand of the ſaid reſolvend, place beyond it the double of the quotient, in form of a diviſor.

· ·
2704(5
25
———
10) . 204

5thly. Dividing the reſolvend, all but the unit's place, by the ſaid diviſor, (which, in this example, is the 10's in 20) ſet down the number of times it goes, (*viz.* 2) both in the quotient, and on the right hand of the diviſor:

· ·
2704(52
25
———
102)204 *Reſolvend.*

6thly. Multiplying the whole diviſor by the figure laſt plac'd in the quotient, ſet down the product under, and ſubſtract it from the reſolvend.

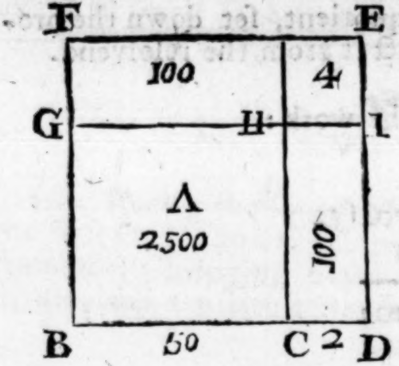
See the work:

```
     . .
    2704(52
    25
    ———
102)204
    204
    ———
      0
```

Note, If the diviſor, multiply'd by the quotient-figure, gives a product greater than the reſolvend, 'tis falſe, and muſt be rectify'd by a ſmaller quotient-figure.

Extraction of the ſquare root being a matter rather lineal than numeral, the reaſon of the rules above given for its operation, will beſt appear from the following figure: Wherein let it be conſider'd, that if the root *B. C.* of the ſquare *A*, be augmented by the magnitude *C. D.* the ſaid ſquare will be enlarged by the figure *C. D. E. F. G. H I.* equal to an oblong or long ſquare, whoſe length ſhall be the root *B. C.* twice taken, with the line *C. D.* and breadth the ſame line *C. D.*

N. B. The particular contents of the ſeveral parts of the figure are as therein ſet down.



The ſeveral Parts of the Square.	2500 100 100 4
Whole Square	2704
Parts of the Root.	50 2
Whole Root	52

In the operation therefore, having taken the neareſt root of the firſt ſquare 27, *viz.* 5, and placed it in the quotient, from the place it ſtands in, (*viz.* the ſecond) it muſt be accounted 50, the ſquare of which, *viz.* 2500 (mark'd *A* in the figure) being ſubſtracted from the given number 2704, there remains 204, equal to the figure *C. D. E. F. G. H. I.* In order to find the content of which, I double the root *B. C. viz* 50, for the two baſes *C. H.* and *H. G.* which making 100, I ſet it down as a diviſor; and to find the height of the ſaid figure, *viz. C.D.* I ſeek how often the ſaid diviſor is contain'd in the number 204, and finding it twice, I ſet down two in the quotient for the ſaid height, and alſo in the unit's place of my diviſor for the baſe *H. I.* Then multiplying the baſes by the height, I have the content of the whole annex'd figure *C. D. E. F. G. H. I. viz.* 204, exactly equal to the number remaining, after the ſubſtraction of the firſt great ſquare 2500; ſo that the number given, *viz.* 2704, appears to be a perfect ſquare, whoſe root or ſide is exactly 52.

Note, As many ſingle ſquares as are in the number propos'd, ſo many times muſt the work of the three laſt rules be repeated; every ſquare being to be brought down, as directed in the third rule.

EXAMPLE.

EXAMPLE.

What's the Root of 582169)763 *Anſwer.*
49

146).921 *Reſolvend.*
876

1523).4569 *Reſolvend.*
4569

0

In this example, to 45 the ſecond remainder, 69 the next ſquare, being brought down, makes 4569 for a new reſolvend; and 76, the quotient-figures, being doubled for a new diviſor, makes 152, which going three times in the reſolvend, 3 is plac'd both in the quotient, and on the right hand of the diviſor, which being then multiply'd by 3, gives 4569, equal to the reſolvend; ſo that nothing remains.

PROOF.

The proof of this extraction, is only multiplying the root found by it ſelf, which, if right, will produce the ſquare number given. *Note*, If any thing remains, it muſt be taken in.

To find the fractional Part of the Root.

Add to the number given, a competent number of pairs of cyphers, and proceed by the foregoing rules: As ſuppoſe 75896 were given, for an example.

EXAMPLE.

```
          ·  ·  · ·  · ·
          75896,000000(275,492
          4
         ----
     47)358
        329
       -----
   545) 2996
        2725
        -----
  5504) 27100
        22016
        ------
 55089) 508400
        495801
        -------
550982) 1259900
        1101964
        -------
         157936
```

Note, As many pairs of cyphers as are added, ſo many places of decimal parts will there be in the root.

To extract the SQUARE ROOT *of a* VULGAR FRACTION.

Find the root of the numerator for a numerator; and let the root of the denominator be its denominator; thus the ſquare root of $\frac{16}{25}$ is $\frac{4}{5}$, and of $\frac{4}{9}$ is $\frac{2}{3}$, *&c.*

The root of a mix'd number is alſo found after the ſame manner, being firſt reduc'd into an improper

per fraction; thus 6 $\frac{1}{4}$, reduc'd, makes $\frac{25}{4}$, the root of which is $\frac{5}{2}$, or 2 $\frac{1}{3}$.

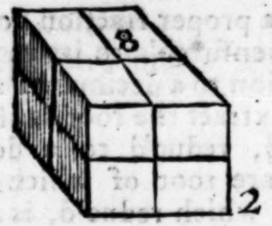
But if either a proper fraction, or a mix'd number, be incommenſurable to its root, to extract it, reduce the fraction to a decimal of an even number of places, and extract the root as if it were a whole number, thus $\frac{7}{8}$, reduc'd to a decimal, will be ,8750, the ſquare root of which is ,935, *&c.* or had it been 4 $\frac{7}{8}$, which reduc'd, is 4,8750, the root would be 2,935, *&c.*

The CUBE ROOT.

EXtraction of the CUBE ROOT, is finding the ſide of a ſolid figure, whoſe length, breadth, and depth, are equal; or, numerically ſpeaking, it is finding what number, multiply'd twice into its ſelf, will produce the number given; thus the Cube Root of 64 is 4, 4 times 4 being 16, and 4 times 16 make 64.

What a Cube is, may be ſomething explain'd by the following figure, wherein 8 dice are ſo diſpos'd, that there are 2 every way, that is, 2 in length, 2 in breadth, and 2 in height; ſo that 8 is the cube, and 2 the root.

See the Figure.



Cube-numbers are either ſingle or compound.

A ſingle cube-number is always leſs than 1000, being produc'd by the multiplication of one ſingle figure, firſt by it ſelf, and then by its product, as 125 from 5, *&c.* ſo that the root of any ſingle cube may be found in the annex'd table; always remembring to take the root of the next leſs cube for any number not there inſerted, as for 513 take 8, or 6 for 220, *&c.*

Cubes	1	8	27	64	125	216	343	512	729
Squares	1	4	9	16	25	36	49	64	81
Roots	1	2	3	4	5	6	7	8	9

A compound cube-number being compos'd by the multiplication of 2 or more figures, firſt by themſelves, and then by their product, always exceeds 1000, as 1728 from 12, or 15625 from 25, *&c.*

If therefore the root is expreſs'd by two figures, its cube muſt at leaſt conſiſt of four; for the leaſt root expreſs'd by two figures is 10, whoſe cube is 1000; if the root has three figures, its cube muſt at leaſt have ſeven, *&c.* So that you cannot augment the root one figure, but you encreaſe the cube three.

Therefore to find the root of any compound cube-number, as ſuppoſe 15625.

1ſt. You muſt diſtinguiſh it into ſingle cubes, by placing dots over every third figure, beginning from the right hand, thus:

$$1\dot{5}62\dot{5}$$

And ſo many dots as happen, ſo many places will the root conſiſt of, which, in this example, are two.

2dly. Drawing a crooked line on the right hand of your number, as in diviſion, ſet down as a quotient the root of your firſt ſingle cube, which, in this example, is 2.

3dly. Placing 8, the cube of 2. the root found, under 15 the firſt ſingle cube, ſubſtract, and to the remainder 7 bring down 625 the next ſingle cube, and it will make 7625, which call a Reſolvend.

See the work:

$$\begin{array}{l} 1\dot{5}62\dot{5}(2 \\ \;8 \\ \hline 7625 \text{ \textit{Reſolvend.}} \end{array}$$

4thly. Draw a line under the reſolvend, and tripling the ſquare of the root 2, ſet the ſaid triple ſquare, (*viz.* 12) under the reſolvend; ſo that units in the ſaid triple ſquare, may ſtand under the place of hundreds in the reſolvend.

5thly. Subſcribe alſo the triple of the root 2 (*viz.* 6) ſo that units, in this, may ſtand under the place of tens in the reſolvend.

6thly. The triple ſquare of the root, and triple root, being plac'd as directed, draw a line under them, and add them together in the order they are plac'd; the ſum 126 is a diviſor.

7thly. Accompting all the reſolvend (except the place of units, (*viz.* 762) a dividend, ſeek how often the diviſor (126) is contain'd in it, and place the number of times (which is here 5) in the quotient.

8thly. Draw a line under the diviſor, and multiplying the triple ſquare (12) by 5, the figure laſt plac'd in the quotient, ſet the product (60) ſo under the ſaid triple ſquare, that units may ſtand under units, and tens under tens.

9thly. Squaring the figure (5) laſt plac'd in the quotient, multiply its ſquare (*viz.* 25) by the triple root (6), and place the product (150) ſo that units, in this, may ſtand under units in the ſaid triple number.

10*thly*. Subſcribe the cube of (5) the figure laſt plac'd in the quotient, which will be 125; ſo that tens in this may ſtand under units in the former product.

11*thly*. Then drawing a line, add the three numbers laſt plac'd together, and ſubſtracting the ſum 7625, (which is call'd the *ablatitium*) from the reſolvend, ſet down the remainder (if any) in order underneath, as in common ſubſtraction.

See the work:

15625(25
8 *Cube of 2.*

7625 *Reſolvend.*

12 *triple Square of 2.*
6 *triple of the Root 2.*

126 *Diviſor.*

60 *triple Square* × 5.
150 *triple Root* × 5 × 5.
125 *Cube of 5.*

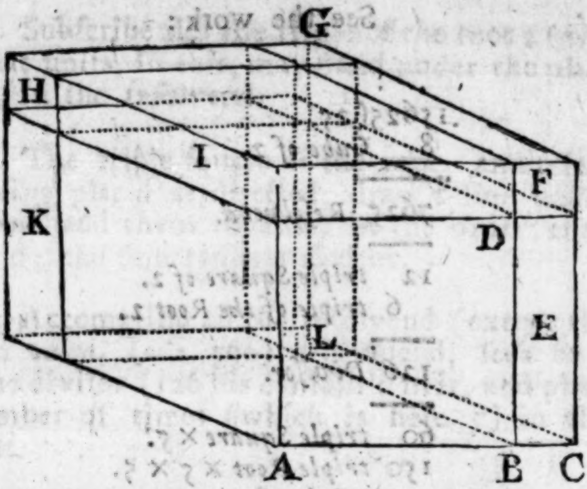
7625 *Ablatitium.*

0

Note, If the given number hath more places, the next ſingle cube muſt be brought down to the laſt remainder, for a new reſolvend, and the work of the 4th, 5th, 6th, 7th, 8th, 9th, 10th, and 11th rules, muſt be repeated as often as you ſo form a new reſolvend.

Note alſo, If the *ablatitium* is greater than the reſolvend, the work is falſe, and muſt be rectify'd by placing a leſſer figure in the quotient.

To account for theſe rules of extracting the cube root, we muſt again make uſe of a figure with literal references, as follows:



From whence it appears, That if the root *A. B.* of the cube *A. D.* be augmented by the magnitude *B. C.* the cube is thereby enlarged by the addition of ſeven parallelepipedes, * *viz.* 1ſt, The little cube *G.* whoſe root is the line *B. C.* 2*d.* The three parallelepipedes mark'd *I. K. E.* which have the ſquare of the root *A. B.* for baſe, and the line *B. C.* for height. 2*d.* The three parallelepipedes mark'd *F. H. L.* which have the ſquare of the line *B. C.* for baſe, and the root *A. B.* for height.

In the operation therefore of the foregoing example, 15625 being divided into ſingle cubes, it appears that the root will conſiſt of two places: The neareſt root then of the firſt cube 15, *viz.* 2, being plac'd in the quotient, where its value will be 20, from the place it ſtands in, and its cube 8000 ſubſtracted from the given number 15625, the remainder 7625 will be equal to the ſeven parallelepipedes *G. I. K. E. F. H. L.* the baſes of three of which, *viz.* *I. K. E.* being the ſquare of the root *A. B.* three times the ſquare of 20, *viz.* 1200 is ſet down under the ſaid remainder, or reſolvend; alſo three more of the ſaid parallelepipedes, *viz.* *F. H. L.* having their height equal to the ſaid root, three times 20, *viz.* 60, the ſum of the three heights, is added to the ſaid 1200, the ſum of the baſes of the other three, for a diviſor; by which, dividing the reſolvend, the number of times it goes, *viz.* 5, is the height of the three parallelepipedes *I. K. E.* and the root of the ſquare of the baſes of the other three parallelepipedes *F. H. L.* and alſo the root or ſide of the parallelepipedon, or little cube *G.* multiply therefore 1200 by 5, for the whole contents of the parallelepipedes *I. K. E.* *viz.* 6000; and ſquaring the ſaid 5 for the baſes of the three parallelepipedes *F. H. L.* multiply their three heights 60, by the ſaid ſquare of their baſes, the product is 1500, their whole contents; under which ſetting down alſo the cube of 5, *viz.* 125, for the content of the parallelepipedon *G.* the ſum of theſe three numbers I find to be juſt 7652, exactly equal to the reſolvend; which ſhews the number given to be a perfect cube, whoſe root is exactly 25.

---

* *Parallelepipedes is a Geometrical Term, ſignifying regular Figures, whoſe oppoſite Sides and Angles are equal.*

To make all more plain, ſee the work at length, with the vacant places fill'd with cyphers.

• •		
15625	———————————	20
8000	*Cube* A. D.	5
7625	*Remainder or Reſolvend*	25 *Root.*
1200	*Baſes of Parallelepipedes* I. K. E.	
60	*Height of Parallelepides* F. H. L.	
1260	*Diviſor.*	
6000	*Content of Parallelepipedes* I. K. E.	
1500	*Content of Parallelepipedes* F. H. L.	
125	*Content of Parallelepipedon* G.	
7625	*Total or Ablatitium.*	
···0		

PROOF.

PROOF.

The proof of this extraction, is by multiplying the root found, firſt by it ſelf, and that product again by the root, adding the remainder, if there be any, to the laſt product; thus,

25 *the Root*
multiply'd by 25
125
50
gives 625
again by 25 *the Root,*
3125
1250
gives 15625

To find the fractional part of the root,

Add to the number given, ſo many tenaries of cyphers, as you would have places of decimal parts in the root, and proceed by the foregoing rules. As ſuppoſe 865635 were given for an example.

EXAMPLE.

EXAMPLE.

865	635	000	000	000	(95,304 *Root.*
729					
136	635	—	—	—	1ſt. *Reſolvend.*
24	3				
	27				
24	57	—	—	—	*Diviſor,*
121	5				
6	75				
	125				
128	375	—	—	—	*Ablatitium.*
8	260	000	—	—	2d. *Reſolvend.*
2	707	5			
	2	85			
2	710	35	—	—	*Diviſor.*
8	122	5			
	25	65			
		27			
8	148	177	—	—	*Ablatitium.*
	111	823	000	—	3d. *Reſolvend.*
	272	462	7		
		28	59		
	272	491	29	—	*Diviſor.*
	111	823	000	000	4th. *Reſolvend.*
	27	246	270		
			285	90	
	27	246	555	90	*Diviſor.*
	108	985	080	0	
		4	574	40	
				64	
	108	989	654	464	*Ablatitium.*
	2	833	345	536	*Remainder.*

In this example, there being three ternaries of cyphers, the root muſt have three decimal places.

Note, The diviſor going no times in the third reſolvend, the ſame figures are immediately brought down again, and with a new ternary of cyphers form the 4th reſolvend.

To extract the Cube Root of a Vulgar Fraction.

The cube root of a vulgar fraction is found after the ſame manner as the ſquare root, *viz.* by finding the root of the numerator given, for a new numerator, and the root of the denominator given, for a new denominator.

Alſo if the vulgar fraction be incommenſurable, it muſt be reduc'd to a decimal, as directed in the ſquare root, and then extracted.

FINIS.

ERRATA.

Page 80. *lin.* 1. *for* 4th, *read* 14th; *and for* 2d. *read* 3d. *pag.* 93. *lin.* 2. *for* 5921 l. *r.* 5291 l. *p.* 115. *l.* 3. *for* 6893 Crowns, *r.* 2256 Crowns. *p.* 154. *l.* 7. *for* $\frac{7}{4}$, *r.* $\frac{4}{7}$. *p.* 155. *l.* 1. *for* $\frac{1}{4}$, *r.* $\frac{4}{1}$. *p.* 157. *l.* 3. *for* Quot. 31 $\frac{3}{4}$, *r.* Quot. 31 $\frac{2}{3}$. *p.* 184. *l.* 4. *for* 1000 miles, *r.* 100 miles.

THE CONTENTS.

PART I. Vulgar Arithmetick.

Chap. I. *Numeration*, Page 3
Chap. I. *Addition*, 4
Chap. I. *Subſtraction*, 7
Chap. I. *Multiplication*, 8
Chap. I. *Diviſion*, 13
Chap. I. *Tables of Coins*, *Weights*, &c. 18
Chap. II. *Reduction*, 22
Chap. III. *The Rule of Three* *Direct*, 25
Chap. III. *The Rule of Three* *Indirect*, 34
Chap. III. *The Rule of Three* *Double*, 36
Chap. IV. *Practice*, 43
Chap. IV. *Trett and Tare*, 57
Chap. V. *Intereſt*, *Simple*, 64
Chap. V. *Intereſt*, *Compound*, 71
Chap. VI. *Rebate or Diſcount*, 75
Chap. VII. *Equation of Payments*, 83
Chap. VIII. *Fellowſhip*, *without Time*, 85
Chap. VIII. *Fellowſhip*, *with Time*, 88
Chap. IX. *Allegation*, *Medial*, 94
Chap. IX. *Allegation*, *Alternate*, 97
Chap. X. *The Rule of Falſe*, *Single Poſition*, 104
Chap. X. *The Rule of Falſe*, *Double Poſition*, 106
Chap. XI. *Exchanges with* France, 110
Chap. XI. *Exchanges with* Italy, 115
Chap. XI. *Exchanges with* Spain, 117
Chap. XI. *Exchanges with* Portugal, 120
Chap. XI. *Exchanges with* Holland, *&c.* 122

The CONTENTS.

Explanation of Characters, Page 126
Chap. XII. *Progreſſion,* *Arithmetical,* 128
Geometrical, 131
Chap. XIII. *Vulgar Fractions,* 139

Part II. Decimal Arithmetick.

Chap. I. *Notation,* Page 162
Addition, 163
Subſtraction, 164
Multiplication, 165
Diviſion, 167
Chap. II. *Reduction,* 173
Decimal Tables of Coin, &c. 180
Chap. III. *The Rule of Three* *Direct,* 181
Indirect, 183
Double, 184
Chap. IV. *Practice,* 185
Chap. V. *Simple Intereſt,* 188
Table ſhewing the Number of Days, &c. 193
Compound Intereſt, 194
The Amount of Annuities, 195
Rebate at Simple Intereſt, 197
Rebate at Compound Intereſt, 200
The preſent Worth of Annuities, 201
Decimal Tables of Intereſt and Rebate, 202
Free Holds, 204
Chap. VI. *Fellowſhip,* *without Time,* 207
with Time, 208
Chap. VII. *Extract. of the* *Square Root,* 210
Cube Root, 217

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